

# Mathematica 11.3 Integration Test Results

## on the problems in "4 Trig functions\4.4 Cotangent"

### Test results for the 52 problems in "4.4.0 (a trig)^m (b cot)^n.m"

Problem 39: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \cot [e + f x])^n (a \sin [e + f x])^m dx$$

Optimal (type 5, 87 leaves, 2 steps):

$$-\frac{1}{b f (1+n)} (b \cot [e + f x])^{1+n} \text{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{1}{2}(1-m+n), \frac{3+n}{2}, \cos [e + f x]^2\right] (a \sin [e + f x])^m (\sin [e + f x]^2)^{\frac{1}{2}(1-m+n)}$$

Result (type 6, 2957 leaves):

$$\begin{aligned} & \left( 2(3+m-n) \text{AppellF1}\left[\frac{1}{2}(1+m-n), -n, 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\ & \quad \left. \cos\left[\frac{1}{2}(e+fx)\right]^2 \cot\left[\frac{1}{2}(e+fx)\right] \cot[e+fx]^n (b \cot[e+fx])^n \sin[e+fx]^m (a \sin[e+fx])^m \right) / \\ & \left( f(1+m-n) \left( -2n \text{AppellF1}\left[\frac{1}{2}(3+m-n), 1-n, 1+m, \frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2(1+m) \text{AppellF1}\left[\frac{1}{2}(3+m-n), -n, 2+m, \frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (3+m-n) \text{AppellF1}\left[\frac{1}{2}(1+m-n), -n, 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\ & \quad \left. - \left( \left( 2(3+m-n)n \text{AppellF1}\left[\frac{1}{2}(1+m-n), -n, 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 \cot\left[\frac{1}{2}(e+fx)\right] \right. \right. \right. \\ & \quad \left. \left. \cot[e+fx]^{-1+n} \sin[e+fx]^{-2+m} \right) / \left( (1+m-n) \left( -2n \text{AppellF1}\left[\frac{1}{2}(3+m-n), 1-n, 1+m, \frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2(1+m) \text{AppellF1}\left[\frac{1}{2}(3+m-n), -n, 2+m, \frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. \left. (3+m-n) \text{AppellF1}\left[\frac{1}{2}(1+m-n), -n, 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \right) + \end{aligned}$$



$$\begin{aligned}
& \cos\left[\frac{1}{2}(e+fx)\right]^2 \cot\left[\frac{1}{2}(e+fx)\right] \cot[e+fx]^n \sin[e+fx]^m \\
& \left( -(3+m-n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m-n), -n, 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& (3+m-n) \cot\left[\frac{1}{2}(e+fx)\right]^2 \left( -\frac{1}{3+m-n}(1+m-n) n \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m-n), 1-n, 1+m, 1+\frac{1}{2}(3+m-n), \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3+m-n}(1+m)(1+m-n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m-n), -n, 2+m, 1+\frac{1}{2}(3+m-n), \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \\
& 2n \left( -\frac{1}{5+m-n}(1+m)(3+m-n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m-n), 1-n, 2+m, 1+\frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m-n}(1-n)(3+m-n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m-n), 2-n, 1+m, 1+\frac{1}{2}(5+m-n), \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \\
& 2(1+m) \left( -\frac{1}{5+m-n}(3+m-n) n \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m-n), 1-n, 2+m, 1+\frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{5+m-n}(2+m)(3+m-n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m-n), -n, 3+m, 1+\frac{1}{2}(5+m-n), \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left( (1+m-n) \left( -2n \operatorname{AppellF1}\left[\frac{1}{2}(3+m-n), 1-n, 1+m, \frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& 2(1+m) \operatorname{AppellF1}\left[\frac{1}{2}(3+m-n), -n, 2+m, \frac{1}{2}(5+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \\
& \left. \left. (3+m-n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m-n), -n, 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right) \right) \Big)
\end{aligned}$$

**Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d \cot[e+fx])^n \sin[e+fx]^2 dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{(d \cot[e+fx])^{1+n} \operatorname{Hypergeometric2F1}\left[2, \frac{1+n}{2}, \frac{3+n}{2}, -\cot[e+fx]^2\right]}{df(1+n)}$$

Result (type 6, 5097 leaves):







$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( n \left( -\frac{1}{5-n} 3(3-n) \operatorname{AppellF1}\left[1+\frac{3-n}{2}, 1-n, 4, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5-n} (1-n)(3-n) \operatorname{AppellF1}\left[1+\frac{3-n}{2}, 2-n, 3, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + 3 \left( -\frac{1}{5-n} (3-n) n \operatorname{AppellF1}\left[1+\frac{3-n}{2}, 1-n, \right. \right. \\
& \quad \left. \left. 4, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{5-n} 4(3-n) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[1+\frac{3-n}{2}, -n, 5, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
& \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, \right. \right. \right. \\
& \quad \left. \left. 3, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. 3 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)
\end{aligned}$$

**Problem 47: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Cot}[e+fx])^n \operatorname{Sin}[e+fx]^4 dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{(d \operatorname{Cot}[e+fx])^{1+n} \operatorname{Hypergeometric2F1}\left[3, \frac{1+n}{2}, \frac{3+n}{2}, -\operatorname{Cot}[e+fx]^2\right]}{df(1+n)}$$

Result (type 6, 8475 leaves):

$$\begin{aligned}
& \left( 2^{5-n} (-3+n) \operatorname{Cot}[e+fx]^{-n} (d \operatorname{Cot}[e+fx])^n \right. \\
& \quad \left( \operatorname{Cos}[4(e+fx)] \left( \frac{1}{16} \operatorname{Cot}[e+fx]^n - \frac{1}{4} i \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)] - \frac{3}{8} \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)]^2 + \right. \right. \\
& \quad \left. \left. \frac{1}{4} i \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)]^3 + \frac{1}{16} \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)]^4 \right) - \frac{1}{16} i \operatorname{Cot}[e+fx]^n \operatorname{Sin}[4(e+fx)] - \right. \\
& \quad \left. \frac{1}{4} \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)] \operatorname{Sin}[4(e+fx)] + \frac{3}{8} i \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)]^2 \operatorname{Sin}[4(e+fx)] + \right. \\
& \quad \left. \frac{1}{4} \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)]^3 \operatorname{Sin}[4(e+fx)] - \frac{1}{16} i \operatorname{Cot}[e+fx]^n \operatorname{Sin}[2(e+fx)]^4 \operatorname{Sin}[4(e+fx)] \right) + \\
& \quad \left. \operatorname{Cos}[2(e+fx)]^4 \left( \frac{1}{16} \operatorname{Cos}[4(e+fx)] \operatorname{Cot}[e+fx]^n - \frac{1}{16} i \operatorname{Cot}[e+fx]^n \operatorname{Sin}[4(e+fx)] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \cos [2 (e+f x)]^3 \left( \cos [4 (e+f x)] \left( -\frac{1}{4} \cot [e+f x]^n + \frac{1}{4} i \cot [e+f x]^n \sin [2 (e+f x)] \right) + \right. \\
& \quad \left. \frac{1}{4} i \cot [e+f x]^n \sin [4 (e+f x)] + \frac{1}{4} \cot [e+f x]^n \sin [2 (e+f x)] \sin [4 (e+f x)] \right) + \\
& \cos [2 (e+f x)]^2 \left( \cos [4 (e+f x)] \left( \frac{3}{8} \cot [e+f x]^n - \frac{3}{4} i \cot [e+f x]^n \sin [2 (e+f x)] - \frac{3}{8} \cot [e+f x]^n \sin [2 (e+f x)]^2 \right) - \right. \\
& \quad \left. \frac{3}{8} i \cot [e+f x]^n \sin [4 (e+f x)] - \frac{3}{4} \cot [e+f x]^n \sin [2 (e+f x)] \sin [4 (e+f x)] + \frac{3}{8} i \cot [e+f x]^n \sin [2 (e+f x)]^2 \sin [4 (e+f x)] \right) + \\
& \cos [2 (e+f x)] \left( \cos [4 (e+f x)] \left( -\frac{1}{4} \cot [e+f x]^n + \frac{3}{4} i \cot [e+f x]^n \sin [2 (e+f x)] + \frac{3}{4} \cot [e+f x]^n \sin [2 (e+f x)]^2 - \frac{1}{4} i \cot [e+f x]^n \right. \right. \\
& \quad \left. \left. \sin [2 (e+f x)]^3 \right) + \frac{1}{4} i \cot [e+f x]^n \sin [4 (e+f x)] + \frac{3}{4} \cot [e+f x]^n \sin [2 (e+f x)] \sin [4 (e+f x)] - \frac{3}{4} i \cot [e+f x]^n \right. \\
& \quad \left. \sin [2 (e+f x)]^2 \sin [4 (e+f x)] - \frac{1}{4} \cot [e+f x]^n \sin [2 (e+f x)]^3 \sin [4 (e+f x)] \right) \left( \cot \left[ \frac{1}{2} (e+f x) \right] - \tan \left[ \frac{1}{2} (e+f x) \right] \right)^n \\
& \tan \left[ \frac{1}{2} (e+f x) \right] \left( - \left( \left( \text{AppellF1} \left[ \frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \right. \right. \\
& \quad \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 3 \text{AppellF1} \left[ \frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right) \left. \right) + \\
& \left( 2 \text{AppellF1} \left[ \frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \quad \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 4, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 4 \text{AppellF1} \left[ \frac{3-n}{2}, -n, 5, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+f x) \right]^2 - \\
& \text{AppellF1} \left[ \frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] / \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 5, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 5 \text{AppellF1} \left[ \frac{3-n}{2}, -n, 6, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right) \left. \right) / \\
& \left( f (-1+n) \left( 1 + \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right)^5 \left( - \frac{1}{(-1+n) \left( 1 + \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right)^6} 5 \times 2^{5-n} (-3+n) \sec \left[ \frac{1}{2} (e+f x) \right]^2 \left( \cot \left[ \frac{1}{2} (e+f x) \right] - \tan \left[ \frac{1}{2} (e+f x) \right] \right)^n \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e+f x) \right]^2 \left( - \left( \left( \text{AppellF1} \left[ \frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right)^2 \right) / \left( (-3+n) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1} \left[ \frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan \left[ \frac{1}{2} (e+f x) \right]^2, -\tan \left[ \frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e+f x) \right]^2 \right) \right) \right) \right)
\end{aligned}$$













$$\begin{aligned}
& -\frac{1}{4fn(-4+n^2)} (d \operatorname{Cot}[e+fx])^n \left( (-2+n)n \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Hypergeometric2F1}\left[-1-\frac{n}{2}, -n, -\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad (2+n) \left( n \operatorname{Hypergeometric2F1}\left[1-\frac{n}{2}, -n, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. 2(-2+n) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Hypergeometric2F1}\left[-n, -\frac{n}{2}, 1-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right) \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-n} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2
\end{aligned}$$

**Problem 50: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Cot}[e+fx])^n \operatorname{Sin}[e+fx] dx$$

Optimal (type 5, 73 leaves, 1 step):

$$\frac{(d \operatorname{Cot}[e+fx])^{1+n} \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \operatorname{Cos}[e+fx]^2\right] \operatorname{Sin}[e+fx] (\operatorname{Sin}[e+fx]^2)^{n/2}}{df(1+n)}$$

Result (type 6, 1973 leaves):

$$\begin{aligned}
& -\left( 4(-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Cot}[e+fx]^n (d \operatorname{Cot}[e+fx])^n \operatorname{Sin}[e+fx] \right) / \\
& \left( f(-2+n) \left( 2n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 2, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 3, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\
& \left( 4(-4+n)n \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Cot}[e+fx]^{-1+n} \operatorname{Csc}[e+fx]^2 \right) / \\
& \quad \left( (-2+n) \left( 2n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 2, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. 4 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 3, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. (-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left( 8(-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^3 \operatorname{Cot}[e+fx]^n \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \quad \left( (-2+n) \left( 2n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 2, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, -n, 2, 2 - \frac{n}{2}, \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \Big) - \left(4(-4+n) \cos\left[\frac{1}{2}(e+fx)\right]^4 \cot[e+fx]^n \right. \\
& \quad \left. \left(-\frac{1}{2 - \frac{n}{2}} \left(1 - \frac{n}{2}\right) n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 2, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{2 - \frac{n}{2}} \right. \right. \\
& \quad \left. \left. 2 \left(1 - \frac{n}{2}\right) \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) \Big) / \left((-2+n) \right. \\
& \quad \left. \left(2 n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 2, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, -n, 2, 2 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2\right)\right) + \\
& \quad \left(4(-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, -n, 2, 2 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^4 \cot[e+fx]^n \left(-(-4+n) \operatorname{AppellF1}\left[ \right. \right. \right. \\
& \quad \left. \left. 1 - \frac{n}{2}, -n, 2, 2 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 + (-4+n) \cot\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \quad \left. \left. \left(-\frac{1}{2 - \frac{n}{2}} \left(1 - \frac{n}{2}\right) n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 2, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \right. \right. \\
& \quad \left. \left. \frac{1}{2 - \frac{n}{2}} 2 \left(1 - \frac{n}{2}\right) \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) + 2 n \right. \\
& \quad \left. \left(-\frac{1}{3 - \frac{n}{2}} 2 \left(2 - \frac{n}{2}\right) \operatorname{AppellF1}\left[3 - \frac{n}{2}, 1 - n, 3, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3 - \frac{n}{2}} \right. \right. \\
& \quad \left. \left. (1 - n) \left(2 - \frac{n}{2}\right) \operatorname{AppellF1}\left[3 - \frac{n}{2}, 2 - n, 2, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) + \\
& \quad 4 \left(-\frac{1}{3 - \frac{n}{2}} \left(2 - \frac{n}{2}\right) n \operatorname{AppellF1}\left[3 - \frac{n}{2}, 1 - n, 3, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \\
& \quad \left. \frac{1}{3 - \frac{n}{2}} 3 \left(2 - \frac{n}{2}\right) \operatorname{AppellF1}\left[3 - \frac{n}{2}, -n, 4, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) \Big) \Big) / \\
& \quad \left((-2+n) \left(2 n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 2, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \operatorname{AppellF1}\left[2 - \frac{n}{2}, \right. \right. \right. \\
& \quad \left. \left. -n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.
\end{aligned}$$







$$\begin{aligned}
& \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right) + 2\left(n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 4\right. \\
& \left.\operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 5, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \\
& \frac{1}{-2+n} 16(-4+n) \cos\left[\frac{1}{2}(e+fx)\right]^6 \cot[e+fx]^n \sin\left[\frac{1}{2}(e+fx)\right]^2 \left(-\left(\operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 3, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) / \left((-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 3, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2\left(n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 3, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. 3 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) - \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right) \\
& \left(-\frac{1}{2-\frac{n}{2}}\left(1-\frac{n}{2}\right) n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 3, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{2-\frac{n}{2}}\right. \\
& \left. 3\left(1-\frac{n}{2}\right) \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) / \left((-4+n)\right. \\
& \left.\operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 3, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2\left(n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 3, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 3 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \\
& \left(-\frac{1}{2-\frac{n}{2}}\left(1-\frac{n}{2}\right) n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \\
& \left.\frac{1}{2-\frac{n}{2}} 4\left(1-\frac{n}{2}\right) \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 5, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) / \\
& \left((-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 4, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \left. 2\left(n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 5, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \left(\operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 3, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \left.\sec\left[\frac{1}{2}(e+fx)\right]^2\right) \left(2\left(n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 3, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. 3 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (-4+n)\right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{2-\frac{n}{2}} \left(1-\frac{n}{2}\right) n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 3, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \\
& \quad \left. \frac{1}{2-\frac{n}{2}} 3 \left(1-\frac{n}{2}\right) \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \\
& 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left( n \left( -\frac{1}{3-\frac{n}{2}} 3 \left(2-\frac{n}{2}\right) \operatorname{AppellF1}\left[3-\frac{n}{2}, 1-n, 4, 4-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3-\frac{n}{2}} (1-n) \left(2-\frac{n}{2}\right) \operatorname{AppellF1}\left[3-\frac{n}{2}, 2-n, 3, 4-\frac{n}{2}, \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + 3 \left( -\frac{1}{3-\frac{n}{2}} \left(2-\frac{n}{2}\right) n \right. \\
& \quad \left. \operatorname{AppellF1}\left[3-\frac{n}{2}, 1-n, 4, 4-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3-\frac{n}{2}} \right. \\
& \quad \left. \left. 4 \left(2-\frac{n}{2}\right) \operatorname{AppellF1}\left[3-\frac{n}{2}, -n, 5, 4-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
& \left( (-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 3, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 3, \right. \right. \right. \\
& \quad \left. \left. 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 3 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \left( \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 4, 2-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
& \left( 2 \left( n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 5, \right. \right. \right. \\
& \quad \left. \left. 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (-4+n) \right. \\
& \quad \left( -\frac{1}{2-\frac{n}{2}} \left(1-\frac{n}{2}\right) n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 4, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \\
& \quad \left. \frac{1}{2-\frac{n}{2}} 4 \left(1-\frac{n}{2}\right) \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 5, 3-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \\
& 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left( n \left( -\frac{1}{3-\frac{n}{2}} 4 \left(2-\frac{n}{2}\right) \operatorname{AppellF1}\left[3-\frac{n}{2}, 1-n, 5, 4-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3-\frac{n}{2}}(1-n)\left(2-\frac{n}{2}\right) \text{AppellF1}\left[3-\frac{n}{2}, 2-n, 4, 4-\frac{n}{2}, \right. \\
& \left. \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) + 4\left(-\frac{1}{3-\frac{n}{2}}\left(2-\frac{n}{2}\right)n \right. \\
& \left. \text{AppellF1}\left[3-\frac{n}{2}, 1-n, 5, 4-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3-\frac{n}{2}} \right. \\
& \left. 5\left(2-\frac{n}{2}\right) \text{AppellF1}\left[3-\frac{n}{2}, -n, 6, 4-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) \Bigg) / \\
& \left( (-4+n) \text{AppellF1}\left[1-\frac{n}{2}, -n, 4, 2-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2\left(n \text{AppellF1}\left[2-\frac{n}{2}, 1-n, \right. \right. \right. \\
& \left. \left. 4, 3-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. 4 \text{AppellF1}\left[2-\frac{n}{2}, -n, 5, 3-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

**Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \cot[e+fx])^n (a \csc[e+fx])^m dx$$

Optimal (type 5, 83 leaves, 1 step):

$$-\frac{1}{bf(1+n)} (b \cot[e+fx])^{1+n} (a \csc[e+fx])^m \text{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{1}{2}(1+m+n), \frac{3+n}{2}, \cos[e+fx]^2\right] (\sin[e+fx]^2)^{\frac{1}{2}(1+m+n)}$$

Result (type 6, 3166 leaves):

$$\begin{aligned}
& -\left( \left( 2(-3+m+n) \text{AppellF1}\left[\frac{1}{2}(1-m-n), -n, 1-m, \frac{1}{2}(3-m-n), \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \right. \\
& \left. \left. \cos\left[\frac{1}{2}(e+fx)\right]^2 \cot\left[\frac{1}{2}(e+fx)\right] \cot[e+fx]^n (b \cot[e+fx])^n \csc[e+fx]^m (a \csc[e+fx])^m \right) \right) / \\
& \left( f(-1+m+n) \left( 2n \text{AppellF1}\left[\frac{1}{2}(3-m-n), 1-n, 1-m, \frac{1}{2}(5-m-n), \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. \left. 2(-1+m) \text{AppellF1}\left[\frac{1}{2}(3-m-n), -n, 2-m, \frac{1}{2}(5-m-n), \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. (-3+m+n) \text{AppellF1}\left[\frac{1}{2}(1-m-n), -n, 1-m, \frac{1}{2}(3-m-n), \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \\
& \left( \left( 2(-3+m+n) \text{AppellF1}\left[\frac{1}{2}(1-m-n), -n, 1-m, \frac{1}{2}(3-m-n), \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 \cot[e+fx]^n \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left( (-1+m+n) \left( 2n \operatorname{AppellF1} \left[ \frac{1}{2} (3-m-n), 1-n, 1-m, \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad 2(-1+m) \operatorname{AppellF1} \left[ \frac{1}{2} (3-m-n), -n, 2-m, \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \\
& \quad \left. \left. (-3+m+n) \operatorname{AppellF1} \left[ \frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \cot \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left( 2(-3+m+n) \operatorname{AppellF1} \left[ \frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right. \\
& \quad \left. \cos \left[ \frac{1}{2} (e+fx) \right]^2 \cot \left[ \frac{1}{2} (e+fx) \right] \cot [e+fx]^n \operatorname{Csc} [e+fx]^m \right. \\
& \quad \left( -(-3+m+n) \operatorname{AppellF1} \left[ \frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \cot \left[ \frac{1}{2} (e+fx) \right] \right. \\
& \quad \left. \operatorname{Csc} \left[ \frac{1}{2} (e+fx) \right]^2 + (-3+m+n) \cot \left[ \frac{1}{2} (e+fx) \right]^2 \left( -\frac{1}{3-m-n} (1-m-n) n \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (1-m-n), 1-n, 1-m, 1 + \frac{1}{2} (3-m-n), \right. \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{3-m-n} (1-m) (1-m-n) \operatorname{AppellF1} \left[ \right. \right. \\
& \quad \left. \left. 1 + \frac{1}{2} (1-m-n), -n, 2-m, 1 + \frac{1}{2} (3-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) \right) + \\
& \quad 2n \left( -\frac{1}{5-m-n} (1-m) (3-m-n) \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (3-m-n), 1-n, 2-m, 1 + \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] + \frac{1}{5-m-n} (1-n) (3-m-n) \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (3-m-n), \right. \right. \\
& \quad \left. \left. 2-n, 1-m, 1 + \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) - \\
& \quad 2(-1+m) \left( -\frac{1}{5-m-n} (3-m-n) n \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (3-m-n), 1-n, 2-m, 1 + \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] - \frac{1}{5-m-n} (2-m) (3-m-n) \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (3-m-n), \right. \right. \\
& \quad \left. \left. -n, 3-m, 1 + \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) \right) \Big) / \\
& \left( (-1+m+n) \left( 2n \operatorname{AppellF1} \left[ \frac{1}{2} (3-m-n), 1-n, 1-m, \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad 2(-1+m) \operatorname{AppellF1} \left[ \frac{1}{2} (3-m-n), -n, 2-m, \frac{1}{2} (5-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \\
& \quad \left. \left. (-3+m+n) \operatorname{AppellF1} \left[ \frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \cot \left[ \frac{1}{2} (e+fx) \right]^2 \right) \right) \right) \Big) \Big)
\end{aligned}$$

Test results for the 61 problems in "4.4.10 (c+d x)^m (a+b cot)^n.m"

### Problem 3: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Cot}[a + b x] dx$$

Optimal (type 4, 53 leaves, 4 steps):

$$-\frac{i x^2}{2} + \frac{x \operatorname{Log}[1 - e^{2i(a+bx)}]}{b} - \frac{i \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{2 b^2}$$

Result (type 4, 166 leaves):

$$\frac{1}{2} x^2 \operatorname{Cot}[a] - \left( \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) / \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

### Problem 7: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{Cot}[a + b x]^2 dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$-\frac{i x^2}{b} - \frac{x^3}{3} - \frac{x^2 \operatorname{Cot}[a + b x]}{b} + \frac{2 x \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^2} - \frac{i \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^3}$$

Result (type 4, 181 leaves):

$$-\frac{x^3}{3} + \frac{x^2 \operatorname{Csc}[a] \operatorname{Csc}[a + b x] \operatorname{Sin}[b x]}{b} - \left( \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) / \left( b^3 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

### Problem 13: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Cot}[a + b x]^3 dx$$

Optimal (type 4, 91 leaves, 7 steps):

$$-\frac{x}{2b} + \frac{i x^2}{2} - \frac{\operatorname{Cot}[a + b x]}{2b^2} - \frac{x \operatorname{Cot}[a + b x]^2}{2b} - \frac{x \operatorname{Log}[1 - e^{2i(a+bx)}]}{b} + \frac{i \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{2b^2}$$

Result (type 4, 201 leaves):

$$-\frac{1}{2} x^2 \operatorname{Cot}[a] - \frac{x \operatorname{Csc}[a + b x]^2}{2b} + \frac{\operatorname{Csc}[a] \operatorname{Csc}[a + b x] \operatorname{Sin}[b x]}{2b^2} +$$

$$\left( \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2i b x}] - \right. \right.$$

$$2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] +$$

$$\left. \left. i \operatorname{PolyLog}[2, e^{2i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right) \operatorname{Tan}[a] \right) \Bigg/ \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

### Problem 37: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b \operatorname{Cot}[e + f x]) dx$$

Optimal (type 4, 147 leaves, 8 steps):

$$\frac{a (c + d x)^4}{4d} - \frac{i b (c + d x)^4}{4d} + \frac{b (c + d x)^3 \operatorname{Log}[1 - e^{2i(e+fx)}]}{f} -$$

$$\frac{3 i b d (c + d x)^2 \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{2f^2} + \frac{3 b d^2 (c + d x) \operatorname{PolyLog}[3, e^{2i(e+fx)}]}{2f^3} + \frac{3 i b d^3 \operatorname{PolyLog}[4, e^{2i(e+fx)}]}{4f^4}$$

Result (type 4, 524 leaves):



$$\begin{aligned}
& -\frac{1}{4 f^3} b c d^2 e^{-i e} \operatorname{Csc}[e] \\
& \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]\right) + 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] \right) - \\
& \frac{1}{4} b d^3 e^{i e} \operatorname{Csc}[e] \left( x^4 + (-1 + e^{-2 i e}) x^4 + \frac{1}{2 f^4} e^{-2 i e} (-1 + e^{2 i e}) \right. \\
& \left. (2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] + 6 f^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 6 i f x \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right]) \right) + \\
& \frac{1}{4} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \operatorname{Csc}[e] (b \operatorname{Cos}[e] + a \operatorname{Sin}[e]) + \frac{b c^3 \operatorname{Csc}[e] (-f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e])}{f (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} - \\
& \left( 3 b c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \left. \left. (i f x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\right. \right. \\
& \left. \left. \operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}\right]) \operatorname{Tan}[e] \right) \right) \Bigg/ \left( 2 f^2 \sqrt{\operatorname{Sec}[e]^2 (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} \right)
\end{aligned}$$

**Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 (a + b \operatorname{Cot}[e + f x]) dx$$

Optimal (type 4, 112 leaves, 7 steps):

$$\frac{a (c + d x)^3}{3 d} - \frac{i b (c + d x)^3}{3 d} + \frac{b (c + d x)^2 \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]}{f} - \frac{i b d (c + d x) \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right]}{f^2} + \frac{b d^2 \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]}{2 f^3}$$

Result (type 4, 361 leaves):

$$\begin{aligned}
& -\frac{1}{12 f^3} b d^2 e^{-i e} \operatorname{Csc}[e] \\
& \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] \right) + 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] \right) + \\
& \frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Csc}[e] \left( b \operatorname{Cos}[e] + a \operatorname{Sin}[e] \right) + \frac{b c^2 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e] \right)}{f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} - \\
& \left( b c d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \left. \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}\right] \right) \operatorname{Tan}[e] \right) \right) \Bigg/ \left( f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
\end{aligned}$$

**Problem 39: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) (a + b \operatorname{Cot}[e + f x]) dx$$

Optimal (type 4, 83 leaves, 6 steps):

$$\frac{a (c + d x)^2}{2 d} - \frac{i b (c + d x)^2}{2 d} + \frac{b (c + d x) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]}{f} - \frac{i b d \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right]}{2 f^2}$$

Result (type 4, 196 leaves):

$$\begin{aligned}
& a c x + \frac{1}{2} a d x^2 + \frac{1}{2} b d x^2 \operatorname{Cot}[e] + \frac{b c \operatorname{Log}[\operatorname{Sin}[e + f x]]}{f} - \\
& \left( b d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \left. \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - \right. \right. \\
& \left. \left. 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]\right] + \right. \right. \\
& \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}\right] \right) \operatorname{Tan}[e] \right) \right) \Bigg/ \left( 2 f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
\end{aligned}$$

**Problem 42: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 (a + b \operatorname{Cot}[e + f x])^2 dx$$

Optimal (type 4, 295 leaves, 15 steps):

$$\begin{aligned}
& - \frac{i b^2 (c + d x)^3}{f} + \frac{a^2 (c + d x)^4}{4 d} - \frac{i a b (c + d x)^4}{2 d} - \frac{b^2 (c + d x)^4}{4 d} - \frac{b^2 (c + d x)^3 \operatorname{Cot}[e + f x]}{f} + \frac{3 b^2 d (c + d x)^2 \operatorname{Log}[1 - e^{2 i (e + f x)}]}{f^2} + \\
& \frac{2 a b (c + d x)^3 \operatorname{Log}[1 - e^{2 i (e + f x)}]}{f} - \frac{3 i b^2 d^2 (c + d x) \operatorname{PolyLog}[2, e^{2 i (e + f x)}]}{f^3} - \frac{3 i a b d (c + d x)^2 \operatorname{PolyLog}[2, e^{2 i (e + f x)}]}{f^2} + \\
& \frac{3 b^2 d^3 \operatorname{PolyLog}[3, e^{2 i (e + f x)}]}{2 f^4} + \frac{3 a b d^2 (c + d x) \operatorname{PolyLog}[3, e^{2 i (e + f x)}]}{f^3} + \frac{3 i a b d^3 \operatorname{PolyLog}[4, e^{2 i (e + f x)}]}{2 f^4}
\end{aligned}$$

Result (type 4, 1313 leaves):

$$\begin{aligned}
& -\frac{1}{4 f^4} b^2 d^3 e^{-i e} \operatorname{Csc}[e] \\
& \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i \left( -1 + e^{2 i e} \right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]\right) + 6 \left( -1 + e^{2 i e} \right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i \left( -1 + e^{2 i e} \right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] \right) - \\
& \frac{1}{2 f^3} a b c d^2 e^{-i e} \operatorname{Csc}[e] \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i \left( -1 + e^{2 i e} \right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]\right) + 6 \left( -1 + e^{2 i e} \right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + \right. \\
& \left. 3 i \left( -1 + e^{2 i e} \right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] \right) - \frac{1}{2} a b d^3 e^{i e} \operatorname{Csc}[e] \left( x^4 + \left( -1 + e^{-2 i e} \right) x^4 + \frac{1}{2 f^4} e^{-2 i e} \left( -1 + e^{2 i e} \right) \right. \\
& \left. \left( 2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] + 6 f^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 6 i f x \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right] \right) \right) + \\
& \frac{3 b^2 c^2 d \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e] \right)}{f^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} + \\
& \frac{2 a b c^3 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e] \right)}{f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} + \\
& \frac{1}{8 f} \operatorname{Csc}[e] \operatorname{Csc}[e+f x] \left( 4 a^2 c^3 f x \operatorname{Cos}[f x] - 4 b^2 c^3 f x \operatorname{Cos}[f x] + 6 a^2 c^2 d f x^2 \operatorname{Cos}[f x] - 6 b^2 c^2 d f x^2 \operatorname{Cos}[f x] + 4 a^2 c d^2 f x^3 \operatorname{Cos}[f x] - \right. \\
& 4 b^2 c d^2 f x^3 \operatorname{Cos}[f x] + a^2 d^3 f x^4 \operatorname{Cos}[f x] - b^2 d^3 f x^4 \operatorname{Cos}[f x] - 4 a^2 c^3 f x \operatorname{Cos}[2 e+f x] + 4 b^2 c^3 f x \operatorname{Cos}[2 e+f x] - \\
& 6 a^2 c^2 d f x^2 \operatorname{Cos}[2 e+f x] + 6 b^2 c^2 d f x^2 \operatorname{Cos}[2 e+f x] - 4 a^2 c d^2 f x^3 \operatorname{Cos}[2 e+f x] + 4 b^2 c d^2 f x^3 \operatorname{Cos}[2 e+f x] - \\
& a^2 d^3 f x^4 \operatorname{Cos}[2 e+f x] + b^2 d^3 f x^4 \operatorname{Cos}[2 e+f x] + 8 b^2 c^3 \operatorname{Sin}[f x] + 24 b^2 c^2 d x \operatorname{Sin}[f x] + 8 a b c^3 f x \operatorname{Sin}[f x] + 24 b^2 c d^2 x^2 \operatorname{Sin}[f x] + \\
& 12 a b c^2 d f x^2 \operatorname{Sin}[f x] + 8 b^2 d^3 x^3 \operatorname{Sin}[f x] + 8 a b c d^2 f x^3 \operatorname{Sin}[f x] + 2 a b d^3 f x^4 \operatorname{Sin}[f x] + 8 a b c^3 f x \operatorname{Sin}[2 e+f x] + \\
& \left. 12 a b c^2 d f x^2 \operatorname{Sin}[2 e+f x] + 8 a b c d^2 f x^3 \operatorname{Sin}[2 e+f x] + 2 a b d^3 f x^4 \operatorname{Sin}[2 e+f x] \right) - \left( 3 b^2 c d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \right. \right. \\
& \left. \left. \frac{1}{\sqrt{1+\operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x+\operatorname{ArcTan}[\operatorname{Tan}[e])}\right] \right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log}\left[\operatorname{Cos}[f x]\right] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}\left[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (f x+\operatorname{ArcTan}[\operatorname{Tan}[e])}\right] \right) \operatorname{Tan}[e] \right) \Bigg) / \\
& \left( f^3 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right) - \left( 3 a b c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1+\operatorname{Tan}[e]^2}} \right. \right. \\
& \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x+\operatorname{ArcTan}[\operatorname{Tan}[e])}\right] + \pi \operatorname{Log}\left[\operatorname{Cos}[f x]\right] + 2 \right. \right. \\
& \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}\left[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (f x+\operatorname{ArcTan}[\operatorname{Tan}[e])}\right] \right) \operatorname{Tan}[e] \right) \Bigg) / \left( f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
\end{aligned}$$

### Problem 43: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^2 (a + b \cot [e + fx])^2 dx$$

Optimal (type 4, 227 leaves, 13 steps):

$$\begin{aligned} & -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} - \frac{b^2(c+dx)^2 \cot [e+fx]}{f} + \frac{2b^2d(c+dx) \operatorname{Log}[1 - e^{2i(e+fx)}]}{f^2} + \\ & \frac{2ab(c+dx)^2 \operatorname{Log}[1 - e^{2i(e+fx)}]}{f} - \frac{ib^2d^2 \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^3} - \frac{2iabd(c+dx) \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^2} + \frac{abd^2 \operatorname{PolyLog}[3, e^{2i(e+fx)}]}{f^3} \end{aligned}$$

Result (type 4, 635 leaves):

$$\begin{aligned} & -\frac{1}{6f^3} ab d^2 e^{-ie} \operatorname{Csc}[e] \\ & \left( 2f^2 x^2 (2e^{2ie} fx + 3i(-1 + e^{2ie}) \operatorname{Log}[1 - e^{2i(e+fx)}]) + 6(-1 + e^{2ie}) fx \operatorname{PolyLog}[2, e^{2i(e+fx)}] + 3i(-1 + e^{2ie}) \operatorname{PolyLog}[3, e^{2i(e+fx)}] \right) + \\ & \frac{1}{3} x (3c^2 + 3cdx + d^2 x^2) \operatorname{Csc}[e] (2ab \operatorname{Cos}[e] + a^2 \operatorname{Sin}[e] - b^2 \operatorname{Sin}[e]) + \\ & \frac{2b^2cd \operatorname{Csc}[e] (-fx \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[fx] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[fx]]) \operatorname{Sin}[e]}{f^2 (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} + \\ & \frac{2abc^2 \operatorname{Csc}[e] (-fx \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[fx] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[fx]]) \operatorname{Sin}[e]}{f (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} + \\ & \frac{\operatorname{Csc}[e] \operatorname{Csc}[e+fx] (b^2c^2 \operatorname{Sin}[fx] + 2b^2cdx \operatorname{Sin}[fx] + b^2d^2x^2 \operatorname{Sin}[fx])}{f} - \left( b^2d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \right. \right. \\ & \left. \left. \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} (ifx(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}[1 + e^{-2ifx}] - 2(fx + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}[1 - e^{2i(fx + \operatorname{ArcTan}[\operatorname{Tan}[e])}]) + \right. \right. \\ & \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[fx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[fx + \operatorname{ArcTan}[\operatorname{Tan}[e]]] + i \operatorname{PolyLog}[2, e^{2i(fx + \operatorname{ArcTan}[\operatorname{Tan}[e])}]) \operatorname{Tan}[e] \right) \right) / \\ & \left( f^3 \sqrt{\operatorname{Sec}[e]^2 (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} \right) - \left( 2abcd \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\ & \left. \left. (ifx(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}[1 + e^{-2ifx}] - 2(fx + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}[1 - e^{2i(fx + \operatorname{ArcTan}[\operatorname{Tan}[e])}]) + \pi \operatorname{Log}[\operatorname{Cos}[fx]] + 2 \right. \right. \\ & \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[fx + \operatorname{ArcTan}[\operatorname{Tan}[e]]] + i \operatorname{PolyLog}[2, e^{2i(fx + \operatorname{ArcTan}[\operatorname{Tan}[e])}]) \operatorname{Tan}[e] \right) \right) / \left( f^2 \sqrt{\operatorname{Sec}[e]^2 (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} \right) \end{aligned}$$

### Problem 47: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^3 (a + b \operatorname{Cot}[e + fx])^3 dx$$

Optimal (type 4, 603 leaves, 28 steps):

$$\begin{aligned} & -\frac{3 i b^3 d (c + dx)^2}{2 f^2} - \frac{3 i a b^2 (c + dx)^3}{f} - \frac{b^3 (c + dx)^3}{2 f} + \frac{a^3 (c + dx)^4}{4 d} - \frac{3 i a^2 b (c + dx)^4}{4 d} - \frac{3 a b^2 (c + dx)^4}{4 d} + \frac{i b^3 (c + dx)^4}{4 d} - \\ & \frac{3 b^3 d (c + dx)^2 \operatorname{Cot}[e + fx]}{2 f^2} - \frac{3 a b^2 (c + dx)^3 \operatorname{Cot}[e + fx]}{f} - \frac{b^3 (c + dx)^3 \operatorname{Cot}[e + fx]^2}{2 f} + \frac{3 b^3 d^2 (c + dx) \operatorname{Log}[1 - e^{2 i (e + fx)}]}{f^3} + \\ & \frac{9 a b^2 d (c + dx)^2 \operatorname{Log}[1 - e^{2 i (e + fx)}]}{f^2} + \frac{3 a^2 b (c + dx)^3 \operatorname{Log}[1 - e^{2 i (e + fx)}]}{f} - \frac{b^3 (c + dx)^3 \operatorname{Log}[1 - e^{2 i (e + fx)}]}{f} - \\ & \frac{3 i b^3 d^3 \operatorname{PolyLog}[2, e^{2 i (e + fx)}]}{2 f^4} - \frac{9 i a b^2 d^2 (c + dx) \operatorname{PolyLog}[2, e^{2 i (e + fx)}]}{f^3} - \frac{9 i a^2 b d (c + dx)^2 \operatorname{PolyLog}[2, e^{2 i (e + fx)}]}{2 f^2} + \\ & \frac{3 i b^3 d (c + dx)^2 \operatorname{PolyLog}[2, e^{2 i (e + fx)}]}{2 f^2} + \frac{9 a b^2 d^3 \operatorname{PolyLog}[3, e^{2 i (e + fx)}]}{2 f^4} + \frac{9 a^2 b d^2 (c + dx) \operatorname{PolyLog}[3, e^{2 i (e + fx)}]}{2 f^3} - \\ & \frac{3 b^3 d^2 (c + dx) \operatorname{PolyLog}[3, e^{2 i (e + fx)}]}{2 f^3} + \frac{9 i a^2 b d^3 \operatorname{PolyLog}[4, e^{2 i (e + fx)}]}{4 f^4} - \frac{3 i b^3 d^3 \operatorname{PolyLog}[4, e^{2 i (e + fx)}]}{4 f^4} \end{aligned}$$

Result (type 4, 2539 leaves):

$$\begin{aligned} & \frac{(-b^3 c^3 - 3 b^3 c^2 dx - 3 b^3 c d^2 x^2 - b^3 d^3 x^3) \operatorname{Csc}[e + fx]^2}{2 f} - \frac{1}{4 f^4} 3 a b^2 d^3 e^{-i e} \operatorname{Csc}[e] \\ & (2 f^2 x^2 (2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}[1 - e^{2 i (e + fx)}]) + 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}[2, e^{2 i (e + fx)}] + 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}[3, e^{2 i (e + fx)}]) - \\ & \frac{1}{4 f^3} 3 a^2 b c d^2 e^{-i e} \operatorname{Csc}[e] (2 f^2 x^2 (2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}[1 - e^{2 i (e + fx)}]) + \\ & 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}[2, e^{2 i (e + fx)}] + 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}[3, e^{2 i (e + fx)}]) + \frac{1}{4 f^3} b^3 c d^2 e^{-i e} \operatorname{Csc}[e] \\ & (2 f^2 x^2 (2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}[1 - e^{2 i (e + fx)}]) + 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}[2, e^{2 i (e + fx)}] + 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}[3, e^{2 i (e + fx)}]) - \\ & \frac{3}{4} a^2 b d^3 e^{i e} \operatorname{Csc}[e] \left( x^4 + (-1 + e^{-2 i e}) x^4 + \frac{1}{2 f^4} e^{-2 i e} (-1 + e^{2 i e}) \right. \\ & \left. (2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}[1 - e^{2 i (e + fx)}] + 6 f^2 x^2 \operatorname{PolyLog}[2, e^{2 i (e + fx)}] + 6 i f x \operatorname{PolyLog}[3, e^{2 i (e + fx)}] - 3 \operatorname{PolyLog}[4, e^{2 i (e + fx)}]) \right) + \\ & \frac{1}{4} b^3 d^3 e^{i e} \operatorname{Csc}[e] \left( x^4 + (-1 + e^{-2 i e}) x^4 + \frac{1}{2 f^4} e^{-2 i e} (-1 + e^{2 i e}) \right. \\ & \left. (2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}[1 - e^{2 i (e + fx)}] + 6 f^2 x^2 \operatorname{PolyLog}[2, e^{2 i (e + fx)}] + 6 i f x \operatorname{PolyLog}[3, e^{2 i (e + fx)}] - 3 \operatorname{PolyLog}[4, e^{2 i (e + fx)}]) \right) + \\ & \frac{3 b^3 c d^2 \operatorname{Csc}[e] (-f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e])}{f^3 (\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)} + \end{aligned}$$

$$\begin{aligned}
& \frac{9 a b^2 c^2 d \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right)}{f^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} + \\
& \frac{3 a^2 b c^3 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right)}{f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} - \\
& \frac{b^3 c^3 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right)}{f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} + \\
& \left( 3 x^2 \left( -a^3 c^2 d + 3 i a^2 b c^2 d + 3 a b^2 c^2 d - i b^3 c^2 d + a^3 c^2 d \operatorname{Cos}[2 e] + 3 i a^2 b c^2 d \operatorname{Cos}[2 e] - 3 a b^2 c^2 d \operatorname{Cos}[2 e] - i b^3 c^2 d \operatorname{Cos}[2 e] + \right. \right. \\
& \quad \left. \left. i a^3 c^2 d \operatorname{Sin}[2 e] - 3 a^2 b c^2 d \operatorname{Sin}[2 e] - 3 i a b^2 c^2 d \operatorname{Sin}[2 e] + b^3 c^2 d \operatorname{Sin}[2 e] \right) \right) / \left( 2 \left( -1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e] \right) \right) + \\
& \left( x^3 \left( -a^3 c d^2 + 3 i a^2 b c d^2 + 3 a b^2 c d^2 - i b^3 c d^2 + a^3 c d^2 \operatorname{Cos}[2 e] + 3 i a^2 b c d^2 \operatorname{Cos}[2 e] - 3 a b^2 c d^2 \operatorname{Cos}[2 e] - i b^3 c d^2 \operatorname{Cos}[2 e] + \right. \right. \\
& \quad \left. \left. i a^3 c d^2 \operatorname{Sin}[2 e] - 3 a^2 b c d^2 \operatorname{Sin}[2 e] - 3 i a b^2 c d^2 \operatorname{Sin}[2 e] + b^3 c d^2 \operatorname{Sin}[2 e] \right) \right) / \left( -1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e] \right) + \\
& \left( x^4 \left( -a^3 d^3 + 3 i a^2 b d^3 + 3 a b^2 d^3 - i b^3 d^3 + a^3 d^3 \operatorname{Cos}[2 e] + 3 i a^2 b d^3 \operatorname{Cos}[2 e] - 3 a b^2 d^3 \operatorname{Cos}[2 e] - i b^3 d^3 \operatorname{Cos}[2 e] + \right. \right. \\
& \quad \left. \left. i a^3 d^3 \operatorname{Sin}[2 e] - 3 a^2 b d^3 \operatorname{Sin}[2 e] - 3 i a b^2 d^3 \operatorname{Sin}[2 e] + b^3 d^3 \operatorname{Sin}[2 e] \right) \right) / \left( 4 \left( -1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e] \right) \right) + \\
& x \left( a^3 c^3 - 3 a b^2 c^3 + \frac{3 i a^2 b c^3}{-1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e]} + \frac{3 i a^2 b c^3 \operatorname{Cos}[2 e] - 3 a^2 b c^3 \operatorname{Sin}[2 e]}{-1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e]} + \right. \\
& \quad \left. \frac{-2 i b^3 c^3 \operatorname{Cos}[2 e] + 2 b^3 c^3 \operatorname{Sin}[2 e]}{-1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e]} \right) \frac{\left( 1 + \operatorname{Cos}[2 e] + \operatorname{Cos}[4 e] + i \operatorname{Sin}[2 e] + i \operatorname{Sin}[4 e] \right)}{\left( -1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e] \right)} + \\
& \quad \frac{-2 i b^3 c^3 \operatorname{Cos}[4 e] + 2 b^3 c^3 \operatorname{Sin}[4 e]}{\left( -1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e] \right) \left( 1 + \operatorname{Cos}[2 e] + \operatorname{Cos}[4 e] + i \operatorname{Sin}[2 e] + i \operatorname{Sin}[4 e] \right)} - \\
& \quad \left. \frac{i b^3 c^3}{-1 + \operatorname{Cos}[6 e] + i \operatorname{Sin}[6 e]} + \frac{-i b^3 c^3 \operatorname{Cos}[6 e] + b^3 c^3 \operatorname{Sin}[6 e]}{-1 + \operatorname{Cos}[6 e] + i \operatorname{Sin}[6 e]} \right) + \frac{1}{2 f^2} \\
& 3 \operatorname{Csc}[e] \operatorname{Csc}[e + f x] \left( b^3 c^2 d \operatorname{Sin}[f x] + 2 a b^2 c^3 f \operatorname{Sin}[f x] + 2 b^3 c d^2 x \operatorname{Sin}[f x] + 6 a b^2 c^2 d f x \operatorname{Sin}[f x] + b^3 d^3 x^2 \operatorname{Sin}[f x] + \right. \\
& \quad \left. 6 a b^2 c d^2 f x^2 \operatorname{Sin}[f x] + 2 a b^2 d^3 f x^3 \operatorname{Sin}[f x] \right) - \left( 3 b^3 d^3 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \quad \left. \left. (i f x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}[1 + e^{-2 i f x}] - 2 (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}]] \right) + \right. \\
& \quad \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}] \right] \operatorname{Tan}[e] \right) \Bigg) / \\
& \left( 2 f^4 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right) - \left( 9 a b^2 c d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \quad \left. \left. (i f x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}[1 + e^{-2 i f x}] - 2 (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}]] \right) + \right. \\
& \quad \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}] \right] \operatorname{Tan}[e] \right) \Bigg) /
\end{aligned}$$

$$\left( f^3 \sqrt{\sec[e]^2 (\cos[e]^2 + \sin[e]^2)} \right) - \left( 9 a^2 b c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\ \left. \left. (i f x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}[1 + e^{-2 i f x}] - 2 (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}]] + \right. \right. \\ \left. \left. \pi \operatorname{Log}[\cos[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\sin[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}]] \operatorname{Tan}[e] \right) \right) / \\ \left( 2 f^2 \sqrt{\sec[e]^2 (\cos[e]^2 + \sin[e]^2)} \right) + \left( 3 b^3 c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\ \left. \left. (i f x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]) - \pi \operatorname{Log}[1 + e^{-2 i f x}] - 2 (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]) \operatorname{Log}[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}]] + \pi \operatorname{Log}[\cos[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\sin[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e])}]] \operatorname{Tan}[e] \right) \right) / \left( 2 f^2 \sqrt{\sec[e]^2 (\cos[e]^2 + \sin[e]^2)} \right)$$

**Problem 48: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 (a + b \operatorname{Cot}[e + f x])^3 dx$$

Optimal (type 4, 433 leaves, 22 steps):

$$\frac{b^3 c d x}{f} - \frac{b^3 d^2 x^2}{2 f} - \frac{3 i a b^2 (c + d x)^2}{f} + \frac{a^3 (c + d x)^3}{3 d} - \frac{i a^2 b (c + d x)^3}{d} - \frac{a b^2 (c + d x)^3}{d} + \frac{i b^3 (c + d x)^3}{3 d} - \\ \frac{b^3 d (c + d x) \operatorname{Cot}[e + f x]}{f^2} - \frac{3 a b^2 (c + d x)^2 \operatorname{Cot}[e + f x]}{f} - \frac{b^3 (c + d x)^2 \operatorname{Cot}[e + f x]^2}{2 f} + \frac{6 a b^2 d (c + d x) \operatorname{Log}[1 - e^{2 i (e + f x)}]}{f^2} + \\ \frac{3 a^2 b (c + d x)^2 \operatorname{Log}[1 - e^{2 i (e + f x)}]}{f} - \frac{b^3 (c + d x)^2 \operatorname{Log}[1 - e^{2 i (e + f x)}]}{f} + \frac{b^3 d^2 \operatorname{Log}[\sin[e + f x]]}{f^3} - \frac{3 i a b^2 d^2 \operatorname{PolyLog}[2, e^{2 i (e + f x)}]}{f^3} - \\ \frac{3 i a^2 b d (c + d x) \operatorname{PolyLog}[2, e^{2 i (e + f x)}]}{f^2} + \frac{i b^3 d (c + d x) \operatorname{PolyLog}[2, e^{2 i (e + f x)}]}{f^2} + \frac{3 a^2 b d^2 \operatorname{PolyLog}[3, e^{2 i (e + f x)}]}{2 f^3} - \frac{b^3 d^2 \operatorname{PolyLog}[3, e^{2 i (e + f x)}]}{2 f^3}$$

Result (type 4, 1825 leaves):

$$- \frac{1}{4 f^3} a^2 b d^2 e^{-i e} \operatorname{Csc}[e] \\ \left( 2 f^2 x^2 (2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}[1 - e^{2 i (e + f x)}]) + 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}[2, e^{2 i (e + f x)}] + 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}[3, e^{2 i (e + f x)}] \right) + \\ \frac{1}{12 f^3} b^3 d^2 e^{-i e} \operatorname{Csc}[e] \left( 2 f^2 x^2 (2 e^{2 i e} f x + 3 i (-1 + e^{2 i e}) \operatorname{Log}[1 - e^{2 i (e + f x)}]) + 6 (-1 + e^{2 i e}) f x \operatorname{PolyLog}[2, e^{2 i (e + f x)}] + \right. \\ \left. 3 i (-1 + e^{2 i e}) \operatorname{PolyLog}[3, e^{2 i (e + f x)}] \right) + \frac{b^3 d^2 \operatorname{Csc}[e] (-f x \cos[e] + \operatorname{Log}[\cos[f x] \sin[e] + \cos[e] \sin[f x]] \sin[e])}{f^3 (\cos[e]^2 + \sin[e]^2)}$$



$$\begin{aligned}
& \frac{6 a b^2 c d \operatorname{Csc}[e] \left(-f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e]\right)}{f^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)} + \\
& \frac{3 a^2 b c^2 \operatorname{Csc}[e] \left(-f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e]\right)}{f \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)} - \\
& \frac{b^3 c^2 \operatorname{Csc}[e] \left(-f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e]\right)}{f \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)} + \\
& \frac{1}{12 f^2} \operatorname{Csc}[e] \operatorname{Csc}[e + f x]^2 \left(6 b^3 c d \operatorname{Cos}[e] + 18 a b^2 c^2 f \operatorname{Cos}[e] + 6 b^3 d^2 x \operatorname{Cos}[e] + 36 a b^2 c d f x \operatorname{Cos}[e] + 18 a^2 b c^2 f^2 x \operatorname{Cos}[e] - 6 b^3 c^2 f^2 x \operatorname{Cos}[e] + \right. \\
& 18 a b^2 d^2 f x^2 \operatorname{Cos}[e] + 18 a^2 b c d f^2 x^2 \operatorname{Cos}[e] - 6 b^3 c d f^2 x^2 \operatorname{Cos}[e] + 6 a^2 b d^2 f^2 x^3 \operatorname{Cos}[e] - 2 b^3 d^2 f^2 x^3 \operatorname{Cos}[e] - 6 b^3 c d \operatorname{Cos}[e + 2 f x] - \\
& 18 a b^2 c^2 f \operatorname{Cos}[e + 2 f x] - 6 b^3 d^2 x \operatorname{Cos}[e + 2 f x] - 36 a b^2 c d f x \operatorname{Cos}[e + 2 f x] - 9 a^2 b c^2 f^2 x \operatorname{Cos}[e + 2 f x] + 3 b^3 c^2 f^2 x \operatorname{Cos}[e + 2 f x] - \\
& 18 a b^2 d^2 f x^2 \operatorname{Cos}[e + 2 f x] - 9 a^2 b c d f^2 x^2 \operatorname{Cos}[e + 2 f x] + 3 b^3 c d f^2 x^2 \operatorname{Cos}[e + 2 f x] - 3 a^2 b d^2 f^2 x^3 \operatorname{Cos}[e + 2 f x] + \\
& b^3 d^2 f^2 x^3 \operatorname{Cos}[e + 2 f x] - 9 a^2 b c^2 f^2 x \operatorname{Cos}[3 e + 2 f x] + 3 b^3 c^2 f^2 x \operatorname{Cos}[3 e + 2 f x] - 9 a^2 b c d f^2 x^2 \operatorname{Cos}[3 e + 2 f x] + \\
& 3 b^3 c d f^2 x^2 \operatorname{Cos}[3 e + 2 f x] - 3 a^2 b d^2 f^2 x^3 \operatorname{Cos}[3 e + 2 f x] + b^3 d^2 f^2 x^3 \operatorname{Cos}[3 e + 2 f x] - 6 b^3 c^2 f \operatorname{Sin}[e] - 12 b^3 c d f x \operatorname{Sin}[e] + \\
& 6 a^3 c^2 f^2 x \operatorname{Sin}[e] - 18 a b^2 c^2 f^2 x \operatorname{Sin}[e] - 6 b^3 d^2 f x^2 \operatorname{Sin}[e] + 6 a^3 c d f^2 x^2 \operatorname{Sin}[e] - 18 a b^2 c d f^2 x^2 \operatorname{Sin}[e] + 2 a^3 d^2 f^2 x^3 \operatorname{Sin}[e] - \\
& 6 a b^2 d^2 f^2 x^3 \operatorname{Sin}[e] + 3 a^3 c^2 f^2 x \operatorname{Sin}[e + 2 f x] - 9 a b^2 c^2 f^2 x \operatorname{Sin}[e + 2 f x] + 3 a^3 c d f^2 x^2 \operatorname{Sin}[e + 2 f x] - 9 a b^2 c d f^2 x^2 \operatorname{Sin}[e + 2 f x] + \\
& a^3 d^2 f^2 x^3 \operatorname{Sin}[e + 2 f x] - 3 a b^2 d^2 f^2 x^3 \operatorname{Sin}[e + 2 f x] - 3 a^3 c^2 f^2 x \operatorname{Sin}[3 e + 2 f x] + 9 a b^2 c^2 f^2 x \operatorname{Sin}[3 e + 2 f x] - 3 a^3 c d f^2 x^2 \operatorname{Sin}[3 e + 2 f x] + \\
& \left. 9 a b^2 c d f^2 x^2 \operatorname{Sin}[3 e + 2 f x] - a^3 d^2 f^2 x^3 \operatorname{Sin}[3 e + 2 f x] + 3 a b^2 d^2 f^2 x^3 \operatorname{Sin}[3 e + 2 f x]\right) - \left(3 a b^2 d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left(e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \right. \right. \\
& \left. \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left(i f x \left(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right) \operatorname{Log}\left[1 - e^{2 i \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right)}\right]\right) + \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right)}\right]\right) \operatorname{Tan}[e]\right) \Bigg) / \\
& \left(f^3 \sqrt{\operatorname{Sec}[e]^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) - \left(3 a^2 b c d \operatorname{Csc}[e] \operatorname{Sec}[e] \left(e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \left. \left. \left(i f x \left(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right) \operatorname{Log}\left[1 - e^{2 i \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right)}\right]\right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right)}\right]\right) \operatorname{Tan}[e]\right) \Bigg) / \\
& \left(f^2 \sqrt{\operatorname{Sec}[e]^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) + \left(b^3 c d \operatorname{Csc}[e] \operatorname{Sec}[e] \left(e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
& \left. \left. \left(i f x \left(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right) \operatorname{Log}\left[1 - e^{2 i \left(f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right)}\right]\right) + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \right.
\end{aligned}$$

$$\left. \text{ArcTan}[\text{Tan}[e]] \text{Log}[\text{Sin}[f x + \text{ArcTan}[\text{Tan}[e]]]] + \text{i PolyLog}\left[2, e^{2i(f x + \text{ArcTan}[\text{Tan}[e]])}\right] \text{Tan}[e] \right) \Bigg/ \left( f^2 \sqrt{\text{Sec}[e]^2 (\text{Cos}[e]^2 + \text{Sin}[e]^2)} \right)$$

**Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{(a + b \text{Cot}[e + f x])^2} dx$$

Optimal (type 4, 839 leaves, 21 steps):

$$\begin{aligned} & -\frac{2 i b^2 (c + d x)^3}{(a^2 + b^2)^2 f} - \frac{2 b^2 (c + d x)^3}{(a - i b) (a + i b)^2 (i a + b - (i a - b) e^{2 i e + 2 i f x}) f} + \frac{(c + d x)^4}{4 (a + i b)^2 d} \\ & - \frac{b (c + d x)^4}{(a + i b)^2 (i a + b) d} - \frac{b^2 (c + d x)^4}{(a^2 + b^2)^2 d} + \frac{3 b^2 d (c + d x)^2 \text{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^2} - \frac{2 b (c + d x)^3 \text{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a - i b) (a + i b)^2 f} \\ & - \frac{2 i b^2 (c + d x)^3 \text{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f} - \frac{3 i b^2 d^2 (c + d x) \text{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^3} - \frac{3 b d (c + d x)^2 \text{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a + i b)^2 (i a + b) f^2} \\ & - \frac{3 b^2 d (c + d x)^2 \text{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^2} + \frac{3 b^2 d^3 \text{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{2 (a^2 + b^2)^2 f^4} - \frac{3 b d^2 (c + d x) \text{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a - i b) (a + i b)^2 f^3} \\ & - \frac{3 i b^2 d^2 (c + d x) \text{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^3} + \frac{3 b d^3 \text{PolyLog}\left[4, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{2 (a + i b)^2 (i a + b) f^4} + \frac{3 b^2 d^3 \text{PolyLog}\left[4, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{2 (a^2 + b^2)^2 f^4} \end{aligned}$$

Result (type 4, 2706 leaves):

$$\begin{aligned} & \frac{1}{2 (a - i b)^3 (a + i b)^2 (-i a (-1 + e^{2 i e}) + b (1 + e^{2 i e})) f^4} \\ & b e^{2 i e} \left( 4 (a - i b) (a + i b) c^2 f^3 (-3 b d + 2 a c f) x - 4 (a - i b) c^2 e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f^3 (-3 b d + 2 a c f) x + \right. \\ & 12 a (a - i b) b c d^2 f^3 x^2 + 12 b^2 (i a + b) c d^2 f^3 x^2 - 12 a (a - i b) b c d^2 e^{-2 i e} f^3 x^2 + 12 b^2 (i a + b) c d^2 e^{-2 i e} f^3 x^2 - \\ & 12 a^2 (a - i b) c^2 d f^4 x^2 - 12 i a (a - i b) b c^2 d f^4 x^2 + 12 a^2 (a - i b) c^2 d e^{-2 i e} f^4 x^2 - 12 i a (a - i b) b c^2 d e^{-2 i e} f^4 x^2 + \\ & 12 (a - i b) (a + i b) c d f^3 (-b d + a c f) x^2 + 4 a (a - i b) b d^3 f^3 x^3 + 4 b^2 (i a + b) d^3 f^3 x^3 - 4 a (a - i b) b d^3 e^{-2 i e} f^3 x^3 + \\ & 4 b^2 (i a + b) d^3 e^{-2 i e} f^3 x^3 - 8 a^2 (a - i b) c d^2 f^4 x^3 - 8 i a (a - i b) b c d^2 f^4 x^3 + 8 a^2 (a - i b) c d^2 e^{-2 i e} f^4 x^3 - 8 i a (a - i b) b c d^2 e^{-2 i e} f^4 x^3 + \\ & 4 (a - i b) (a + i b) d^2 f^3 (-b d + 2 a c f) x^3 - 2 a^2 (a - i b) d^3 f^4 x^4 + 2 a (a - i b) (a + i b) d^3 f^4 x^4 - 2 i a (a - i b) b d^3 f^4 x^4 + \\ & 2 a^2 (a - i b) d^3 e^{-2 i e} f^4 x^4 - 2 i a (a - i b) b d^3 e^{-2 i e} f^4 x^4 - 3 (a - i b) b c^2 d e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) \\ & \left. f^2 \left( 4 f x - 2 \text{ArcTan}\left[\frac{2 a b e^{2 i (e + f x)}}{a^2 (-1 + e^{2 i (e + f x)}) - b^2 (1 + e^{2 i (e + f x)})}\right] + i \text{Log}\left[a^2 (-1 + e^{2 i (e + f x)})^2 + b^2 (1 + e^{2 i (e + f x)})^2\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& 2 a (a - i b) c^3 e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f^3 \\
& \left( 4 f x - 2 \operatorname{ArcTan}\left[\frac{2 a b e^{2 i (e+f x)}}{a^2 (-1 + e^{2 i (e+f x)}) - b^2 (1 + e^{2 i (e+f x)})}\right] + i \operatorname{Log}\left[a^2 (-1 + e^{2 i (e+f x)})^2 + b^2 (1 + e^{2 i (e+f x)})^2\right] \right) - \\
& 6 b (i a + b) c d^2 e^{-2 i e} (-i a (-1 + e^{2 i e}) + b (1 + e^{2 i e})) f \left( 2 f x \left( f x + i \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + \\
& 6 a (a - i b) c^2 d e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f^2 \left( 2 f x \left( f x + i \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) - \\
& b (i a + b) d^3 e^{-2 i e} (-i a (-1 + e^{2 i e}) + b (1 + e^{2 i e})) \left( 2 f^2 x^2 \left( 2 f x + 3 i \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + \right. \\
& \quad \left. 6 f x \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] + 3 i \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + 2 a (a - i b) c d^2 e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f \\
& \left( 2 f^2 x^2 \left( 2 f x + 3 i \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + 6 f x \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] + 3 i \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) + \\
& a (a - i b) d^3 e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) \left( 2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] + \right. \\
& \quad \left. 6 f^2 x^2 \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] + 6 i f x \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] - 3 \operatorname{PolyLog}\left[4, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b}\right] \right) \Big) + \\
& \frac{3 x^2 (-a c^2 d - i b c^2 d + a c^2 d \operatorname{Cos}[2 e] - i b c^2 d \operatorname{Cos}[2 e] + i a c^2 d \operatorname{Sin}[2 e] + b c^2 d \operatorname{Sin}[2 e])}{2 (a - i b) (a + i b) (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e])} + \\
& \frac{x^3 (-a c d^2 - i b c d^2 + a c d^2 \operatorname{Cos}[2 e] - i b c d^2 \operatorname{Cos}[2 e] + i a c d^2 \operatorname{Sin}[2 e] + b c d^2 \operatorname{Sin}[2 e])}{(a - i b) (a + i b) (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e])} + \\
& \frac{x^4 (-a d^3 - i b d^3 + a d^3 \operatorname{Cos}[2 e] - i b d^3 \operatorname{Cos}[2 e] + i a d^3 \operatorname{Sin}[2 e] + b d^3 \operatorname{Sin}[2 e])}{4 (a - i b) (a + i b) (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e])} + \\
& x \\
& \left( \frac{c^3}{(a^2 - 2 i a b - b^2 + a^2 \operatorname{Cos}[4 e] + 2 i a b \operatorname{Cos}[4 e] - b^2 \operatorname{Cos}[4 e] + i a^2 \operatorname{Sin}[4 e] - 2 a b \operatorname{Sin}[4 e] - i b^2 \operatorname{Sin}[4 e])} + \right. \\
& \quad \left( (-i a - b - i a \operatorname{Cos}[2 e] + b \operatorname{Cos}[2 e] + a \operatorname{Sin}[2 e] + i b \operatorname{Sin}[2 e]) (4 a b c^3 \operatorname{Cos}[2 e] + 4 i a b c^3 \operatorname{Sin}[2 e]) \right) / \\
& \quad \left( (a - i b) (a + i b) (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e]) \right. \\
& \quad \left. (a^2 - 2 i a b - b^2 + a^2 \operatorname{Cos}[4 e] + 2 i a b \operatorname{Cos}[4 e] - b^2 \operatorname{Cos}[4 e] + i a^2 \operatorname{Sin}[4 e] - 2 a b \operatorname{Sin}[4 e] - i b^2 \operatorname{Sin}[4 e]) \right) + \\
& \quad \left. (c^3 \operatorname{Cos}[4 e] + i c^3 \operatorname{Sin}[4 e]) / (a^2 - 2 i a b - b^2 + a^2 \operatorname{Cos}[4 e] + 2 i a b \operatorname{Cos}[4 e] - b^2 \operatorname{Cos}[4 e] + i a^2 \operatorname{Sin}[4 e] - 2 a b \operatorname{Sin}[4 e] - i b^2 \operatorname{Sin}[4 e]) \right) + \\
& \frac{b^2 c^3 \operatorname{Sin}[f x] + 3 b^2 c^2 d x \operatorname{Sin}[f x] + 3 b^2 c d^2 x^2 \operatorname{Sin}[f x] + b^2 d^3 x^3 \operatorname{Sin}[f x]}{(a - i b) (a + i b) f (b \operatorname{Cos}[e] + a \operatorname{Sin}[e]) (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x])}
\end{aligned}$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + dx)^2}{(a + b \operatorname{Cot}[e + fx])^2} dx$$

Optimal (type 4, 650 leaves, 18 steps):

$$\begin{aligned} & - \frac{2 i b^2 (c + dx)^2}{(a^2 + b^2)^2 f} - \frac{2 b^2 (c + dx)^2}{(a - i b) (a + i b)^2 (i a + b - (i a - b) e^{2 i e + 2 i f x}) f} + \frac{(c + dx)^3}{3 (a + i b)^2 d} \\ & - \frac{4 b (c + dx)^3}{3 (a + i b)^2 (i a + b) d} - \frac{4 b^2 (c + dx)^3}{3 (a^2 + b^2)^2 d} + \frac{2 b^2 d (c + dx) \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^2} - \frac{2 b (c + dx)^2 \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a - i b) (a + i b)^2 f} \\ & - \frac{2 i b^2 (c + dx)^2 \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f} - \frac{i b^2 d^2 \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^3} - \frac{2 b d (c + dx) \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a + i b)^2 (i a + b) f^2} \\ & - \frac{2 b^2 d (c + dx) \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^2} - \frac{b d^2 \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a - i b) (a + i b)^2 f^3} - \frac{i b^2 d^2 \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^3} \end{aligned}$$

Result (type 4, 1309 leaves):

$$\begin{aligned}
& \frac{1}{3 (a^2 + b^2)^2 (-i a (-1 + e^{2i e}) + b (1 + e^{2i e})) f^3} \\
& b \left( f \left( -12 a b c d e^{2i e} f x - 12 i b^2 c d e^{2i e} f x + 12 a^2 c^2 e^{2i e} f^2 x + 12 i a b c^2 e^{2i e} f^2 x - 6 a b d^2 e^{2i e} f x^2 - 6 i b^2 d^2 e^{2i e} f x^2 + 12 a^2 c d e^{2i e} f^2 x^2 + \right. \right. \\
& \quad 12 i a b c d e^{2i e} f^2 x^2 + 4 a^2 d^2 e^{2i e} f^2 x^3 + 4 i a b d^2 e^{2i e} f^2 x^3 - 6 c (a (-1 + e^{2i e}) + i b (1 + e^{2i e})) (-b d + a c f) \text{ArcTan} \left[ \frac{2 a b e^{2i (e+f x)}}{a^2 (-1 + e^{2i (e+f x)}) - b^2 (1 + e^{2i (e+f x)})} \right] + 6 i d (a (-1 + e^{2i e}) + i b (1 + e^{2i e})) x (-b d + a f (2 c + d x)) \text{Log} \left[ 1 - \frac{(a + i b) e^{2i (e+f x)}}{a - i b} \right] + \\
& \quad 3 i a b c d \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] + 3 b^2 c d \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] - \\
& \quad 3 i a b c d e^{2i e} \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] + 3 b^2 c d e^{2i e} \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] - \\
& \quad 3 i a^2 c^2 f \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] - 3 a b c^2 f \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] + \\
& \quad \left. 3 i a^2 c^2 e^{2i e} f \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] - 3 a b c^2 e^{2i e} f \text{Log} [a^2 (-1 + e^{2i (e+f x)})^2 + b^2 (1 + e^{2i (e+f x)})^2] \right) + \\
& 3 d (a (-1 + e^{2i e}) + i b (1 + e^{2i e})) (-b d + 2 a f (c + d x)) \text{PolyLog} \left[ 2, \frac{(a + i b) e^{2i (e+f x)}}{a - i b} \right] + \\
& 3 i a d^2 (a (-1 + e^{2i e}) + i b (1 + e^{2i e})) \text{PolyLog} \left[ 3, \frac{(a + i b) e^{2i (e+f x)}}{a - i b} \right] + \\
& (3 a^2 c^2 f x \text{Cos} [f x] - 3 b^2 c^2 f x \text{Cos} [f x] + 3 a^2 c d f x^2 \text{Cos} [f x] - 3 b^2 c d f x^2 \text{Cos} [f x] + a^2 d^2 f x^3 \text{Cos} [f x] - \\
& \quad b^2 d^2 f x^3 \text{Cos} [f x] - 3 a^2 c^2 f x \text{Cos} [2 e + f x] - 3 b^2 c^2 f x \text{Cos} [2 e + f x] - \\
& \quad 3 a^2 c d f x^2 \text{Cos} [2 e + f x] - 3 b^2 c d f x^2 \text{Cos} [2 e + f x] - a^2 d^2 f x^3 \text{Cos} [2 e + f x] - \\
& \quad b^2 d^2 f x^3 \text{Cos} [2 e + f x] + 6 b^2 c^2 \text{Sin} [f x] + 12 b^2 c d x \text{Sin} [f x] - 6 a b c^2 f x \text{Sin} [f x] + \\
& \quad 6 b^2 d^2 x^2 \text{Sin} [f x] - 6 a b c d f x^2 \text{Sin} [f x] - 2 a b d^2 f x^3 \text{Sin} [f x]) / \\
& (6 (a - i b) (a + i b) f (b \text{Cos} [e] + a \text{Sin} [e]) (b \text{Cos} [e + f x] + a \text{Sin} [e + f x]))
\end{aligned}$$

**Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{c + d x}{(a + b \text{Cot} [e + f x])^2} dx$$

Optimal (type 4, 213 leaves, 5 steps):

$$\begin{aligned}
& -\frac{(c + d x)^2}{2 (a^2 + b^2) d} + \frac{(b d - 2 a c f - 2 a d f x)^2}{4 a (a - i b)^2 (a + i b) d f^2} + \frac{b (c + d x)}{(a^2 + b^2) f (a + b \text{Cot} [e + f x])} + \\
& \frac{b (b d - 2 a c f - 2 a d f x) \text{Log} \left[ 1 - \frac{(a + i b) e^{2i (e+f x)}}{a - i b} \right]}{(a^2 + b^2)^2 f^2} + \frac{i a b d \text{PolyLog} \left[ 2, \frac{(a + i b) e^{2i (e+f x)}}{a - i b} \right]}{(a^2 + b^2)^2 f^2}
\end{aligned}$$

Result (type 4, 730 leaves):

$$\begin{aligned}
& - \frac{(e + f x) (-2 d e + 2 c f + d (e + f x)) \operatorname{Csc}[e + f x]^2 (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x])^2}{2 (-i a + b) (i a + b) f^2 (a + b \operatorname{Cot}[e + f x])^2} + \\
& \frac{b d \operatorname{Csc}[e + f x]^2 (-a (e + f x) + b \operatorname{Log}[b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x]]) (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x])^2}{(-i a + b) (i a + b) (a^2 + b^2) f^2 (a + b \operatorname{Cot}[e + f x])^2} + \\
& \left( \frac{2 a d e \operatorname{Csc}[e + f x]^2 (-a (e + f x) + b \operatorname{Log}[b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x]]) (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x])^2}{((-i a + b) (i a + b) (a^2 + b^2) f^2 (a + b \operatorname{Cot}[e + f x])^2)} - \right. \\
& \left. \frac{2 a c \operatorname{Csc}[e + f x]^2 (-a (e + f x) + b \operatorname{Log}[b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x]]) (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x])^2}{((-i a + b) (i a + b) (a^2 + b^2) f (a + b \operatorname{Cot}[e + f x])^2)} + \right. \\
& \left. \left( d \operatorname{Csc}[e + f x]^2 \left( e^{i \operatorname{ArcTan}\left[\frac{b}{a}\right]} (e + f x)^2 + \frac{1}{a \sqrt{1 + \frac{b^2}{a^2}}} b \left( i (e + f x) \left( -\pi + 2 \operatorname{ArcTan}\left[\frac{b}{a}\right] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i (e + f x)}\right] - 2 \left( e + f x + \operatorname{ArcTan}\left[\frac{b}{a}\right] \right) \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[1 - e^{2 i (e + f x + \operatorname{ArcTan}\left[\frac{b}{a}\right])}\right] + \pi \operatorname{Log}\left[\operatorname{Cos}[e + f x]\right] + 2 \operatorname{ArcTan}\left[\frac{b}{a}\right] \operatorname{Log}\left[\operatorname{Sin}\left[e + f x + \operatorname{ArcTan}\left[\frac{b}{a}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (e + f x + \operatorname{ArcTan}\left[\frac{b}{a}\right])}\right] \right) \right) \\
& \left. \frac{(b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x])^2}{((-i a + b) (i a + b) \sqrt{\frac{a^2 + b^2}{a^2}} f^2 (a + b \operatorname{Cot}[e + f x])^2)} + \right. \\
& \left. \frac{(\operatorname{Csc}[e + f x]^2 (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x]) (-b d e \operatorname{Sin}[e + f x] + b c f \operatorname{Sin}[e + f x] + b d (e + f x) \operatorname{Sin}[e + f x]))}{((-i a + b) (i a + b) f^2 (a + b \operatorname{Cot}[e + f x])^2)} \right) /
\end{aligned}$$

## Test results for the 23 problems in "4.4.1.2 (d csc)^m (a+b cot)^n.m"

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]^3}{i + \operatorname{Cot}[x]} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$i \operatorname{ArcTanh}[\cos [x]] - \operatorname{Csc}[x]$$

Result (type 3, 26 leaves):

$$-\operatorname{Csc}[x] + i \left( \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] \right)$$

**Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[x]^5}{i + \operatorname{Cot}[x]} dx$$

Optimal (type 3, 28 leaves, 3 steps):

$$\frac{1}{2} i \operatorname{ArcTanh}[\cos [x]] + \frac{1}{2} i \operatorname{Cot}[x] \operatorname{Csc}[x] - \frac{\operatorname{Csc}[x]^3}{3}$$

Result (type 3, 67 leaves):

$$\frac{1}{24} i \operatorname{Csc}[x]^3 \left( 8 i + 9 \left( \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] \right) \sin[x] + 6 \sin[2x] - 3 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] \sin[3x] + 3 \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] \sin[3x] \right)$$

**Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[x]^7}{i + \operatorname{Cot}[x]} dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{3}{8} i \operatorname{ArcTanh}[\cos [x]] + \frac{3}{8} i \operatorname{Cot}[x] \operatorname{Csc}[x] + \frac{1}{4} i \operatorname{Cot}[x] \operatorname{Csc}[x]^3 - \frac{\operatorname{Csc}[x]^5}{5}$$

Result (type 3, 99 leaves):

$$\frac{1}{640} i \operatorname{Csc}[x]^5 \left( 128 i + 150 \left( \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] \right) \sin[x] + 140 \sin[2x] - 75 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] \sin[3x] + 75 \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] \sin[3x] - 30 \sin[4x] + 15 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] \sin[5x] - 15 \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] \sin[5x] \right)$$

**Problem 15: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin [x]^2}{a + b \operatorname{Cot}[x]} dx$$

Optimal (type 3, 72 leaves, 7 steps):

$$\frac{a (a^2 + 3 b^2) x}{2 (a^2 + b^2)^2} - \frac{b^3 \operatorname{Log}[b \operatorname{Cos}[x] + a \operatorname{Sin}[x]]}{(a^2 + b^2)^2} - \frac{(b + a \operatorname{Cot}[x]) \operatorname{Sin}[x]^2}{2 (a^2 + b^2)}$$

Result (type 3, 94 leaves):

$$\frac{1}{4 (a^2 + b^2)^2} \left( 2 a^3 x + 6 a b^2 x - 4 i b^3 x + 4 i b^3 \operatorname{ArcTan}[\operatorname{Tan}[x]] + b (a^2 + b^2) \operatorname{Cos}[2 x] - 2 b^3 \operatorname{Log}[(b \operatorname{Cos}[x] + a \operatorname{Sin}[x])^2] - a^3 \operatorname{Sin}[2 x] - a b^2 \operatorname{Sin}[2 x] \right)$$

## Test results for the 19 problems in "4.4.1.3 (d cos)^m (a+b cot)^n.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x]}{i + \operatorname{Cot}[x]} dx$$

Optimal (type 3, 18 leaves, 8 steps):

$$-i \operatorname{ArcTanh}[\operatorname{Sin}[x]] - \operatorname{Cos}[x] + i \operatorname{Sin}[x]$$

Result (type 3, 44 leaves):

$$-\operatorname{Cos}[x] + i \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Sin}[x] \right)$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x]^3}{i + \operatorname{Cot}[x]} dx$$

Optimal (type 3, 22 leaves, 8 steps):

$$\frac{1}{2} i \operatorname{ArcTanh}[\operatorname{Sin}[x]] + \operatorname{Sec}[x] - \frac{1}{2} i \operatorname{Sec}[x] \operatorname{Tan}[x]$$

Result (type 3, 48 leaves):

$$-\frac{1}{2} i \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Sec}[x] (2 i + \operatorname{Tan}[x]) \right)$$

Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cos}[x]^4}{a + b \operatorname{Cot}[x]} dx$$



Optimal (type 3, 126 leaves, 8 steps):

$$\frac{a (3 a^4 - 6 a^2 b^2 - b^4) x}{8 (a^2 + b^2)^3} - \frac{a^4 b \operatorname{Log}[b \operatorname{Cos}[x] + a \operatorname{Sin}[x]]}{(a^2 + b^2)^3} + \frac{(4 b (2 a^2 + b^2) + a (5 a^2 + b^2) \operatorname{Cot}[x]) \operatorname{Sin}[x]^2}{8 (a^2 + b^2)^2} - \frac{(b + a \operatorname{Cot}[x]) \operatorname{Sin}[x]^4}{4 (a^2 + b^2)}$$

Result (type 3, 179 leaves):

$$\frac{1}{32 (a^2 + b^2)^3} \left( 12 a^5 x - 32 i a^4 b x - 24 a^3 b^2 x - 4 a b^4 x + 32 i a^4 b \operatorname{ArcTan}[\operatorname{Tan}[x]] - 4 b (3 a^4 + 4 a^2 b^2 + b^4) \operatorname{Cos}[2 x] - a^4 b \operatorname{Cos}[4 x] - 2 a^2 b^3 \operatorname{Cos}[4 x] - b^5 \operatorname{Cos}[4 x] - 16 a^4 b \operatorname{Log}[(b \operatorname{Cos}[x] + a \operatorname{Sin}[x])^2] + 8 a^5 \operatorname{Sin}[2 x] + 8 a^3 b^2 \operatorname{Sin}[2 x] + a^5 \operatorname{Sin}[4 x] + 2 a^3 b^2 \operatorname{Sin}[4 x] + a b^4 \operatorname{Sin}[4 x] \right)$$

**Problem 13: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[x]^2}{a + b \operatorname{Cot}[x]} dx$$

Optimal (type 3, 73 leaves, 7 steps):

$$\frac{a (a^2 - b^2) x}{2 (a^2 + b^2)^2} - \frac{a^2 b \operatorname{Log}[b \operatorname{Cos}[x] + a \operatorname{Sin}[x]]}{(a^2 + b^2)^2} + \frac{(b + a \operatorname{Cot}[x]) \operatorname{Sin}[x]^2}{2 (a^2 + b^2)}$$

Result (type 3, 82 leaves):

$$\frac{1}{4 (a^2 + b^2)^2} \left( 4 i a^2 b \operatorname{ArcTan}[\operatorname{Tan}[x]] - b (a^2 + b^2) \operatorname{Cos}[2 x] + a \left( 2 (a - i b)^2 x - 2 a b \operatorname{Log}[(b \operatorname{Cos}[x] + a \operatorname{Sin}[x])^2] + (a^2 + b^2) \operatorname{Sin}[2 x] \right) \right)$$

**Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]^3}{a + b \operatorname{Cot}[x]} dx$$

Optimal (type 3, 79 leaves, 9 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[x]]}{2 a} + \frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[x]]}{a^3} + \frac{b \sqrt{a^2 + b^2} \operatorname{ArcTanh}\left[\frac{a \operatorname{Cos}[x] - b \operatorname{Sin}[x]}{\sqrt{a^2 + b^2}}\right]}{a^3} - \frac{b \operatorname{Sec}[x]}{a^2} + \frac{\operatorname{Sec}[x] \operatorname{Tan}[x]}{2 a}$$

Result (type 3, 192 leaves):

$$-\frac{1}{4a^3} \left( 8b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{-a + b \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right] + \operatorname{Sec}[x]^2 \left( 4ab \operatorname{Cos}[x] + a^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] - \operatorname{Sin} \left[ \frac{x}{2} \right] \right] + 2b^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] - \operatorname{Sin} \left[ \frac{x}{2} \right] \right] + (a^2 + 2b^2) \operatorname{Cos}[2x] \right. \right. \\ \left. \left. \left( \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] - \operatorname{Sin} \left[ \frac{x}{2} \right] \right] - \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] + \operatorname{Sin} \left[ \frac{x}{2} \right] \right] \right) - a^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] + \operatorname{Sin} \left[ \frac{x}{2} \right] \right] - 2b^2 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] + \operatorname{Sin} \left[ \frac{x}{2} \right] \right] - 2a^2 \operatorname{Sin}[x] \right) \right)$$

**Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]}{1 + 2 \operatorname{Cot}[x]} dx$$

Optimal (type 3, 25 leaves, 6 steps):

$$\frac{2 \operatorname{ArcTanh} \left[ \frac{\operatorname{Cos}[x] - 2 \operatorname{Sin}[x]}{\sqrt{5}} \right]}{\sqrt{5}} + \operatorname{ArcTanh}[\operatorname{Sin}[x]]$$

Result (type 3, 57 leaves):

$$\frac{4 \operatorname{ArcTanh} \left[ \frac{1 - 2 \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{5}} \right]}{\sqrt{5}} - \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] - \operatorname{Sin} \left[ \frac{x}{2} \right] \right] + \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{x}{2} \right] + \operatorname{Sin} \left[ \frac{x}{2} \right] \right]$$

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## Test results for the 106 problems in "4.4.2.1 (a+b cot)^m (c+d cot)^n.m"

**Problem 1: Result more than twice size of optimal antiderivative.**

$$\int (a + i a \operatorname{Cot}[c + d x])^n dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{i (a + i a \operatorname{Cot}[c + d x])^n \operatorname{Hypergeometric2F1} \left[ 1, n, 1 + n, \frac{1}{2} (1 + i \operatorname{Cot}[c + d x]) \right]}{2 d n}$$

Result (type 5, 112 leaves):

$$\frac{1}{4 d n (1 + n)} i (1 + i \operatorname{Cot}[c + d x])^{-n} (a + i a \operatorname{Cot}[c + d x])^n \\ \left( 2 (1 + n) (-1 + (1 + i \operatorname{Cot}[c + d x])^n) + n (1 + i \operatorname{Cot}[c + d x])^{1+n} \operatorname{Hypergeometric2F1} \left[ 1, 1 + n, 2 + n, \frac{1}{2} (1 + i \operatorname{Cot}[c + d x]) \right] \right)$$

### Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \cot [x]^2 \sqrt{1 + \cot [x]} \, dx$$

Optimal (type 3, 223 leaves, 12 steps):

$$-\sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\cot [x]}}{\sqrt{2(-1+\sqrt{2})}}\right] + \sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\cot [x]}}{\sqrt{2(-1+\sqrt{2})}}\right] -$$

$$\frac{2}{3}(1+\cot [x])^{3/2} + \frac{\operatorname{Log}\left[1+\sqrt{2}+\cot [x]-\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot [x]}\right]}{2\sqrt{2(1+\sqrt{2})}} - \frac{\operatorname{Log}\left[1+\sqrt{2}+\cot [x]+\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot [x]}\right]}{2\sqrt{2(1+\sqrt{2})}}$$

Result (type 3, 69 leaves):

$$-i\sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot [x]}}{\sqrt{1-i}}\right] + i\sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot [x]}}{\sqrt{1+i}}\right] - \frac{2}{3}(1+\cot [x])^{3/2}$$

### Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \cot [x] \sqrt{1 + \cot [x]} \, dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\sqrt{\frac{1}{2}(-1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{4-3\sqrt{2}+(2-\sqrt{2})\cot [x]}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot [x]}}\right] + \sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTanh}\left[\frac{4+3\sqrt{2}+(2+\sqrt{2})\cot [x]}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot [x]}}\right] - 2\sqrt{1+\cot [x]}$$

Result (type 3, 61 leaves):

$$\sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot [x]}}{\sqrt{1-i}}\right] + \sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot [x]}}{\sqrt{1+i}}\right] - 2\sqrt{1+\cot [x]}$$

### Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \cot [x]^2 (1 + \cot [x])^{3/2} \, dx$$

Optimal (type 3, 139 leaves, 8 steps):

$$-\sqrt{-1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{3-2\sqrt{2}+(1-\sqrt{2})\cot[x]}{\sqrt{2(-7+5\sqrt{2})}\sqrt{1+\cot[x]}}\right] - \sqrt{1+\sqrt{2}} \operatorname{ArcTanh}\left[\frac{3+2\sqrt{2}+(1+\sqrt{2})\cot[x]}{\sqrt{2(7+5\sqrt{2})}\sqrt{1+\cot[x]}}\right] + 2\sqrt{1+\cot[x]} - \frac{2}{5}(1+\cot[x])^{5/2}$$

Result (type 3, 96 leaves):

$$\frac{1}{(\cos[x] + \sin[x])^2} \sin[x] \left( -2 \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1-i}}\right]}{\sqrt{1-i}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1+i}}\right]}{\sqrt{1+i}} \right) (1+\cot[x])^2 \sin[x] - \frac{2}{5} (1+\cot[x])^{5/2} (-5+2\cot[x] + \csc[x]^2) \sin[x] \right)$$

**Problem 44: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[x] (1+\cot[x])^{3/2} dx$$

Optimal (type 3, 221 leaves, 14 steps):

$$-\sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\cot[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] + \sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\cot[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] - 2\sqrt{1+\cot[x]} - \frac{2}{3}(1+\cot[x])^{3/2} - \frac{\operatorname{Log}\left[1+\sqrt{2}+\cot[x]-\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot[x]}\right]}{2\sqrt{1+\sqrt{2}}} + \frac{\operatorname{Log}\left[1+\sqrt{2}+\cot[x]+\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot[x]}\right]}{2\sqrt{1+\sqrt{2}}}$$

Result (type 3, 98 leaves):

$$\frac{1}{(\cos[x] + \sin[x])^2} \sin[x] \left( (1+i) \left( -i\sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1-i}}\right] + \sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1+i}}\right] \right) (1+\cot[x])^2 \sin[x] - \frac{2}{3} (1+\cot[x])^{3/2} (4+\cot[x]) (\cos[x] + \sin[x]) \right)$$

**Problem 45: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[x]^2}{\sqrt{1 + \text{Cot}[x]}} dx$$

Optimal (type 3, 214 leaves, 12 steps):

$$-\frac{1}{2} \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2})} - 2\sqrt{1 + \text{Cot}[x]}}{\sqrt{2(-1 + \sqrt{2})}}\right] + \frac{1}{2} \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2})} + 2\sqrt{1 + \text{Cot}[x]}}{\sqrt{2(-1 + \sqrt{2})}}\right] -$$

$$2\sqrt{1 + \text{Cot}[x]} - \frac{\text{Log}\left[1 + \sqrt{2} + \text{Cot}[x] - \sqrt{2(1 + \sqrt{2})}\sqrt{1 + \text{Cot}[x]}\right]}{4\sqrt{1 + \sqrt{2}}} + \frac{\text{Log}\left[1 + \sqrt{2} + \text{Cot}[x] + \sqrt{2(1 + \sqrt{2})}\sqrt{1 + \text{Cot}[x]}\right]}{4\sqrt{1 + \sqrt{2}}}$$

Result (type 3, 67 leaves):

$$\frac{1}{2} (1 - i)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right] + \frac{1}{2} (1 + i)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right] - 2\sqrt{1 + \text{Cot}[x]}$$

**Problem 46: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[x]}{\sqrt{1 + \text{Cot}[x]}} dx$$

Optimal (type 3, 121 leaves, 5 steps):

$$\frac{1}{2} \sqrt{-1 + \sqrt{2}} \text{ArcTan}\left[\frac{3 - 2\sqrt{2} + (1 - \sqrt{2})\text{Cot}[x]}{\sqrt{2(-7 + 5\sqrt{2})}\sqrt{1 + \text{Cot}[x]}}\right] + \frac{1}{2} \sqrt{1 + \sqrt{2}} \text{ArcTanh}\left[\frac{3 + 2\sqrt{2} + (1 + \sqrt{2})\text{Cot}[x]}{\sqrt{2(7 + 5\sqrt{2})}\sqrt{1 + \text{Cot}[x]}}\right]$$

Result (type 3, 51 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right]}{\sqrt{1 - i}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right]}{\sqrt{1 + i}}$$

### Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cot}[x]^2}{(1 + \text{Cot}[x])^{3/2}} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\frac{1}{2} \sqrt{\frac{1}{2}(-1 + \sqrt{2})} \text{ArcTan}\left[\frac{4 - 3\sqrt{2} + (2 - \sqrt{2})\text{Cot}[x]}{2\sqrt{-7 + 5\sqrt{2}}\sqrt{1 + \text{Cot}[x]}}\right] + \frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{2})} \text{ArcTanh}\left[\frac{4 + 3\sqrt{2} + (2 + \sqrt{2})\text{Cot}[x]}{2\sqrt{7 + 5\sqrt{2}}\sqrt{1 + \text{Cot}[x]}}\right] + \frac{1}{\sqrt{1 + \text{Cot}[x]}}$$

Result (type 3, 65 leaves):

$$\frac{1}{2} \sqrt{1 - i} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right] + \frac{1}{2} \sqrt{1 + i} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right] + \frac{1}{\sqrt{1 + \text{Cot}[x]}}$$

### Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cot}[x]}{(1 + \text{Cot}[x])^{3/2}} dx$$

Optimal (type 3, 226 leaves, 13 steps):

$$\begin{aligned} & \frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2}) - 2\sqrt{1 + \text{Cot}[x]}}}{\sqrt{2(-1 + \sqrt{2})}}\right] - \frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2}) + 2\sqrt{1 + \text{Cot}[x]}}}{\sqrt{2(-1 + \sqrt{2})}}\right] - \\ & \frac{1}{\sqrt{1 + \text{Cot}[x]}} - \frac{\text{Log}[1 + \sqrt{2} + \text{Cot}[x] - \sqrt{2(1 + \sqrt{2})}\sqrt{1 + \text{Cot}[x]}}{4\sqrt{2(1 + \sqrt{2})}} + \frac{\text{Log}[1 + \sqrt{2} + \text{Cot}[x] + \sqrt{2(1 + \sqrt{2})}\sqrt{1 + \text{Cot}[x]}}{4\sqrt{2(1 + \sqrt{2})}} \end{aligned}$$

Result (type 3, 71 leaves):

$$\frac{1}{2} i \sqrt{1 - i} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right] - \frac{1}{2} i \sqrt{1 + i} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right] - \frac{1}{\sqrt{1 + \text{Cot}[x]}}$$

**Problem 49: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[x]^2}{(1 + \text{Cot}[x])^{5/2}} dx$$

Optimal (type 3, 143 leaves, 8 steps):

$$\frac{1}{4} \sqrt{-1 + \sqrt{2}} \text{ArcTan}\left[\frac{3 - 2\sqrt{2} + (1 - \sqrt{2}) \text{Cot}[x]}{\sqrt{2(-7 + 5\sqrt{2})} \sqrt{1 + \text{Cot}[x]}}\right] + \frac{1}{4} \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{3 + 2\sqrt{2} + (1 + \sqrt{2}) \text{Cot}[x]}{\sqrt{2(7 + 5\sqrt{2})} \sqrt{1 + \text{Cot}[x]}}\right] + \frac{1}{3(1 + \text{Cot}[x])^{3/2}} - \frac{1}{\sqrt{1 + \text{Cot}[x]}}$$

Result (type 3, 75 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right]}{2\sqrt{1 - i}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right]}{2\sqrt{1 + i}} + \frac{-2 - 3 \text{Cot}[x]}{3(1 + \text{Cot}[x])^{3/2}}$$

**Problem 50: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[x]}{(1 + \text{Cot}[x])^{5/2}} dx$$

Optimal (type 3, 216 leaves, 13 steps):

$$\frac{1}{4} \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2})} - 2\sqrt{1 + \text{Cot}[x]}}{\sqrt{2(-1 + \sqrt{2})}}\right] - \frac{1}{4} \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2})} + 2\sqrt{1 + \text{Cot}[x]}}{\sqrt{2(-1 + \sqrt{2})}}\right] - \frac{1}{3(1 + \text{Cot}[x])^{3/2}} + \frac{\text{Log}\left[1 + \sqrt{2} + \text{Cot}[x] - \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \text{Cot}[x]}\right]}{8\sqrt{1 + \sqrt{2}}} - \frac{\text{Log}\left[1 + \sqrt{2} + \text{Cot}[x] + \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \text{Cot}[x]}\right]}{8\sqrt{1 + \sqrt{2}}}$$

Result (type 3, 69 leaves):

$$-\frac{1}{4} (1 - i)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right] - \frac{1}{4} (1 + i)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right] - \frac{1}{3(1 + \text{Cot}[x])^{3/2}}$$

**Problem 75: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \cot [c + d x])^{7/2}}{(a + b \cot [c + d x])^2} dx$$

Optimal (type 3, 437 leaves, 16 steps):

$$\frac{a^{5/2} (3 a^2 + 7 b^2) e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c + d x]}}{\sqrt{a} \sqrt{e}}\right]}{b^{5/2} (a^2 + b^2)^2 d} + \frac{(a^2 - 2 a b - b^2) e^{7/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d} -$$

$$\frac{(a^2 - 2 a b - b^2) e^{7/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d} - \frac{(3 a^2 + 2 b^2) e^3 \sqrt{e \cot [c + d x]}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot [c + d x])^{3/2}}{b (a^2 + b^2) d (a + b \cot [c + d x])} +$$

$$\frac{(a^2 + 2 a b - b^2) e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] - \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d} - \frac{(a^2 + 2 a b - b^2) e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] + \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d}$$

Result (type 3, 775 leaves):



$$\begin{aligned}
& \frac{1}{d (a + b \cot [c + d x])^2} \\
& (e \cot [c + d x])^{7/2} \sec [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^2 \left( -\frac{2}{b^2} - \frac{a^3 \sin [c + d x]}{b^2 (-i a + b) (i a + b) (b \cos [c + d x] + a \sin [c + d x])} \right) \tan [c + d x] - \\
& \frac{1}{2 (a - i b) (a + i b) b^2 d \cot [c + d x]^{7/2} (a + b \cot [c + d x])^2} (e \cot [c + d x])^{7/2} \csc [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^2 \\
& \left( -\frac{2 (3 a^3 + 3 a b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \csc [c + d x]^3 \sec [c + d x]}{\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} - \left( a b^2 \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \right. \\
& \left. \left. + \frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \right. \\
& \left. \left. (a + b) \left( \log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \sec [c + d x] \right) / \\
& (2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) - \frac{1}{4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} \\
& b^3 (a + b \cot [c + d x]) \csc [c + d x]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} (-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \\
& \left. (a - b) \left( \log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \sec [c + d x]^2 \sin [2 (c + d x)] \Big)
\end{aligned}$$

**Problem 76:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \cot [c + d x])^{5/2}}{(a + b \cot [c + d x])^2} dx$$

Optimal (type 3, 393 leaves, 15 steps):

$$\begin{aligned}
& - \frac{a^{3/2} (a^2 + 5 b^2) e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot[c+dx]}}{\sqrt{a} \sqrt{e}}\right]}{b^{3/2} (a^2 + b^2)^2 d} - \frac{(a^2 + 2 a b - b^2) e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d} + \\
& \frac{(a^2 + 2 a b - b^2) e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{a^2 e^2 \sqrt{e \cot[c+dx]}}{b (a^2 + b^2) d (a + b \cot[c+dx])} + \\
& \frac{(a^2 - 2 a b - b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot[c+dx] - \sqrt{2} \sqrt{e \cot[c+dx]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d} - \frac{(a^2 - 2 a b - b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot[c+dx] + \sqrt{2} \sqrt{e \cot[c+dx]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d}
\end{aligned}$$

Result (type 3, 731 leaves):

$$\begin{aligned}
& \frac{a^2 (e \cot[c+dx])^{5/2} \operatorname{Sec}[c+dx] (b \cos[c+dx] + a \sin[c+dx]) \tan[c+dx]}{b (-i a + b) (i a + b) d (a + b \cot[c+dx])^2} + \\
& \frac{1}{2 (a - i b) (a + i b) b d \cot[c+dx]^{5/2} (a + b \cot[c+dx])^2} (e \cot[c+dx])^{5/2} \operatorname{Csc}[c+dx]^2 (b \cos[c+dx] + a \sin[c+dx])^2 \\
& \left( - \frac{2 (a^2 + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot[c+dx]}}{\sqrt{a}}\right] (a + b \cot[c+dx]) \operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]}{\sqrt{a} \sqrt{b} (1 + \cot[c+dx])^2 (b + a \tan[c+dx])} - \left( b^2 \cos[2(c+dx)] (a + b \cot[c+dx]) \operatorname{Csc}[c+dx]^3 \right. \right. \\
& \left. \left. + \frac{4 (a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot[c+dx]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} \left( 2 (a - b) \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]}\right] - 2 (a - b) \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]}\right] \right) + \right. \right. \\
& \left. \left. (a + b) \left( \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] \right) \right) \operatorname{Sec}[c+dx] \right) / \\
& \left( 2 (a^2 + b^2) (-1 + \cot[c+dx])^2 (1 + \cot[c+dx])^2 (b + a \tan[c+dx]) \right) + \frac{1}{4 (a^2 + b^2) (1 + \cot[c+dx])^2 (b + a \tan[c+dx])} \\
& a b (a + b \cot[c+dx]) \operatorname{Csc}[c+dx]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot[c+dx]}}{\sqrt{a}}\right] + \sqrt{2} \left( -2 (a + b) \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]}\right] + 2 (a + b) \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]}\right] \right) + \right. \\
& \left. (a - b) \left( \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] \right) \right) \operatorname{Sec}[c+dx]^2 \sin[2(c+dx)] \right)
\end{aligned}$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(e \operatorname{Cot}[c + d x])^{3/2} (a + b \operatorname{Cot}[c + d x])^2} dx$$

Optimal (type 3, 437 leaves, 16 steps):

$$\frac{b^{5/2} (7 a^2 + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c + d x]}}{\sqrt{a} \sqrt{e}}\right]}{a^{5/2} (a^2 + b^2)^2 d e^{3/2}} - \frac{(a^2 + 2 a b - b^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d e^{3/2}} +$$

$$\frac{(a^2 + 2 a b - b^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d e^{3/2}} + \frac{2 a^2 + 3 b^2}{a^2 (a^2 + b^2) d e \sqrt{e \operatorname{Cot}[c + d x]}} - \frac{b^2}{a (a^2 + b^2) d e \sqrt{e \operatorname{Cot}[c + d x]} (a + b \operatorname{Cot}[c + d x])} +$$

$$\frac{(a^2 - 2 a b - b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d e^{3/2}} - \frac{(a^2 - 2 a b - b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d e^{3/2}}$$

Result (type 3, 773 leaves):

$$\begin{aligned}
& \frac{\text{Cot}[c + dx]^2 \text{Csc}[c + dx]^2 (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])^2 \left( \frac{b^3 \text{Sin}[c + dx]}{a^2 (a^2 + b^2) (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])} + \frac{2 \text{Tan}[c + dx]}{a^2} \right)}{d (e \text{Cot}[c + dx])^{3/2} (a + b \text{Cot}[c + dx])^2} - \\
& \frac{1}{2 a^2 (-i a + b) (i a + b) d (e \text{Cot}[c + dx])^{3/2} (a + b \text{Cot}[c + dx])^2} \text{Cot}[c + dx]^{3/2} \text{Csc}[c + dx]^2 (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])^2 \\
& \left( -\frac{2 (3 a^2 b + 3 b^3) \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{\text{Cot}[c + dx]}}{\sqrt{a}}\right] (a + b \text{Cot}[c + dx]) \text{Csc}[c + dx]^3 \text{Sec}[c + dx]}{\sqrt{a} \sqrt{b} (1 + \text{Cot}[c + dx])^2 (b + a \text{Tan}[c + dx])} + \left( a^2 b \text{Cos}[2 (c + dx)] (a + b \text{Cot}[c + dx]) \text{Csc}[c + dx]^3 \right. \right. \\
& \left. \left. \frac{4 (a^2 - b^2) \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{\text{Cot}[c + dx]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \text{ArcTan}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]}] - 2 (a - b) \text{ArcTan}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]}]) + \right. \right. \\
& \left. \left. (a + b) \left( \text{Log}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] - \text{Log}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] \right) \right) \text{Sec}[c + dx] \right) / \\
& \left( 2 (a^2 + b^2) (-1 + \text{Cot}[c + dx])^2 (1 + \text{Cot}[c + dx])^2 (b + a \text{Tan}[c + dx]) \right) - \frac{1}{4 (a^2 + b^2) (1 + \text{Cot}[c + dx])^2 (b + a \text{Tan}[c + dx])} \\
& a^3 (a + b \text{Cot}[c + dx]) \text{Csc}[c + dx]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{\text{Cot}[c + dx]}}{\sqrt{a}}\right] + \sqrt{2} (-2 (a + b) \text{ArcTan}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]}] + 2 (a + b) \text{ArcTan}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]}]) + \right. \\
& \left. (a - b) \left( \text{Log}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] - \text{Log}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] \right) \right) \text{Sec}[c + dx]^2 \text{Sin}[2 (c + dx)] \left. \right)
\end{aligned}$$

**Problem 81: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \text{Cot}[c + dx])^{9/2}}{(a + b \text{Cot}[c + dx])^3} dx$$

Optimal (type 3, 529 leaves, 17 steps):

$$\begin{aligned}
& \frac{a^{5/2} (15 a^4 + 46 a^2 b^2 + 63 b^4) e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{a} \sqrt{e}}\right]}{4 b^{7/2} (a^2 + b^2)^3 d} + \frac{(a-b) (a^2 + 4 a b + b^2) e^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} - \\
& \frac{(a-b) (a^2 + 4 a b + b^2) e^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} - \frac{(15 a^4 + 31 a^2 b^2 + 8 b^4) e^4 \sqrt{e \operatorname{Cot}[c+dx]}}{4 b^3 (a^2 + b^2)^2 d} + \frac{a^2 e^2 (e \operatorname{Cot}[c+dx])^{5/2}}{2 b (a^2 + b^2) d (a + b \operatorname{Cot}[c+dx])^2} + \\
& \frac{a^2 (5 a^2 + 13 b^2) e^3 (e \operatorname{Cot}[c+dx])^{3/2}}{4 b^2 (a^2 + b^2)^2 d (a + b \operatorname{Cot}[c+dx])} - \frac{(a+b) (a^2 - 4 a b + b^2) e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d} + \\
& \frac{(a+b) (a^2 - 4 a b + b^2) e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d}
\end{aligned}$$

Result (type 3, 897 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \cot [c + d x])^3} (e \cot [c + d x])^{9/2} \sec [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \left( -\frac{5 a^4 + 8 a^2 b^2 + 4 b^4}{2 b^3 (-i a + b)^2 (i a + b)^2} + \right. \\
& \left. \frac{a^4}{2 b (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \frac{-5 a^5 \sin [c + d x] - 17 a^3 b^2 \sin [c + d x]}{4 b^3 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) \tan [c + d x] - \\
& \frac{1}{8 (a - i b)^2 (a + i b)^2 b^3 d \cot [c + d x]^{9/2} (a + b \cot [c + d x])^3} (e \cot [c + d x])^{9/2} \csc [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
& \left( -\frac{2 (15 a^5 + 31 a^3 b^2 + 16 a b^4) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}}\right] (a + b \cot [c + d x]) \csc [c + d x]^3 \sec [c + d x]}{\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} - \right. \\
& \frac{1}{(a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} 4 a b^4 \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \\
& \left. \left( \frac{4 (a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\cot [c + d x]}] - 2 (a - b) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \right. \\
& \left. \left. (a + b) \left( \log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \right) \sec [c + d x] - \\
& \frac{1}{4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} (-4 a^2 b^3 + 4 b^5) (a + b \cot [c + d x]) \csc [c + d x]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}}\right] + \sqrt{2} (-2 (a + b) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \\
& \left. (a - b) \left( \log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \right) \sec [c + d x]^2 \sin [2 (c + d x)] \Bigg)
\end{aligned}$$

**Problem 82: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \cot [c + d x])^{7/2}}{(a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 476 leaves, 16 steps):

$$\begin{aligned}
& - \frac{a^{3/2} (3a^4 + 6a^2b^2 + 35b^4) e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{a} \sqrt{e}}\right]}{4b^{5/2} (a^2 + b^2)^3 d} + \frac{(a+b) (a^2 - 4ab + b^2) e^{7/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} - \\
& \frac{(a+b) (a^2 - 4ab + b^2) e^{7/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{a^2 e^2 (e \operatorname{Cot}[c+dx])^{3/2}}{2b (a^2 + b^2) d (a + b \operatorname{Cot}[c+dx])^2} + \\
& \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \operatorname{Cot}[c+dx]}}{4b^2 (a^2 + b^2)^2 d (a + b \operatorname{Cot}[c+dx])} + \frac{(a-b) (a^2 + 4ab + b^2) e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}\right]}{2\sqrt{2} (a^2 + b^2)^3 d} - \\
& \frac{(a-b) (a^2 + 4ab + b^2) e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}\right]}{2\sqrt{2} (a^2 + b^2)^3 d}
\end{aligned}$$

Result (type 3, 870 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \cot [c + d x])^3} (e \cot [c + d x])^{7/2} \sec [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \left( \frac{a^3}{2 b^2 (-i a + b)^2 (i a + b)^2} - \right. \\
& \left. \frac{a^3}{2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \frac{a^4 \sin [c + d x] + 13 a^2 b^2 \sin [c + d x]}{4 b^2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) + \\
& \frac{1}{8 (a - i b)^2 (a + i b)^2 b^2 d \cot [c + d x]^{7/2} (a + b \cot [c + d x])^3} (e \cot [c + d x])^{7/2} \csc [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
& \left( - \frac{2 (3 a^4 + 7 a^2 b^2 + 4 b^4) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \csc [c + d x]^3 \sec [c + d x]}{\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} - \right. \\
& \left( (-4 a^2 b^2 + 4 b^4) \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \\
& \left. \left( \frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \right. \\
& \left. \left. (a + b) \left( \log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \sec [c + d x] \right) / \\
& \left( 2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x]) \right) + \frac{1}{(a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} \\
& 2 a b^3 (a + b \cot [c + d x]) \csc [c + d x]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} (-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \\
& \left. (a - b) \left( \log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \sec [c + d x]^2 \sin [2 (c + d x)] \left. \right)
\end{aligned}$$



**Problem 83: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \cot [c + d x])^{5/2}}{(a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 470 leaves, 16 steps):

$$\begin{aligned} & - \frac{\sqrt{a} (a^4 + 18 a^2 b^2 - 15 b^4) e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c + d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 b^{3/2} (a^2 + b^2)^3 d} - \frac{(a - b) (a^2 + 4 a b + b^2) e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \\ & \frac{(a - b) (a^2 + 4 a b + b^2) e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{a^2 e^2 \sqrt{e \cot [c + d x]}}{2 b (a^2 + b^2) d (a + b \cot [c + d x])^2} - \\ & \frac{a (a^2 + 9 b^2) e^2 \sqrt{e \cot [c + d x]}}{4 b (a^2 + b^2)^2 d (a + b \cot [c + d x])} + \frac{(a + b) (a^2 - 4 a b + b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] - \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d} - \\ & \frac{(a + b) (a^2 - 4 a b + b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] + \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d} \end{aligned}$$

Result (type 3, 864 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \cot [c + d x])^3} (e \cot [c + d x])^{5/2} \csc [c + d x] \sec [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \left( -\frac{a^2}{2 b (-i a + b)^2 (i a + b)^2} + \right. \\
& \left. \frac{a^2 b}{2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} - \frac{3 (-a^3 \sin [c + d x] + 3 a b^2 \sin [c + d x])}{4 b (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) + \\
& \frac{1}{8 (a - i b)^2 (a + i b)^2 b d \cot [c + d x]^{5/2} (a + b \cot [c + d x])^3} (e \cot [c + d x])^{5/2} \csc [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
& \left( -\frac{2 (a^3 + a b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \csc [c + d x]^3 \sec [c + d x]}{\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} - \right. \\
& \frac{1}{(a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} 4 a b^2 \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \\
& \left. \left( \frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \right. \\
& \left. \left. (a + b) \left( \operatorname{Log} [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \operatorname{Log} [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \right) \sec [c + d x] - \\
& \frac{1}{4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} (-4 a^2 b + 4 b^3) (a + b \cot [c + d x]) \csc [c + d x]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} (-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \\
& \left. (a - b) \left( \operatorname{Log} [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \operatorname{Log} [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \right) \sec [c + d x]^2 \sin [2 (c + d x)] \Big)
\end{aligned}$$

**Problem 84:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \cot [c + d x])^{3/2}}{(a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 461 leaves, 16 steps):

$$\begin{aligned}
& - \frac{(3a^4 - 26a^2b^2 + 3b^4) e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{e\cot[c+dx]}}{\sqrt{a}\sqrt{e}}\right]}{4\sqrt{a}\sqrt{b}(a^2+b^2)^3d} - \frac{(a+b)(a^2-4ab+b^2) e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{e\cot[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2}(a^2+b^2)^3d} + \\
& \frac{(a+b)(a^2-4ab+b^2) e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{e\cot[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2}(a^2+b^2)^3d} - \frac{ae\sqrt{e\cot[c+dx]}}{2(a^2+b^2)d(a+b\cot[c+dx])^2} - \\
& \frac{(3a^2-5b^2)e\sqrt{e\cot[c+dx]}}{4(a^2+b^2)^2d(a+b\cot[c+dx])} - \frac{(a-b)(a^2+4ab+b^2) e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e}\cot[c+dx] - \sqrt{2}\sqrt{e\cot[c+dx]}\right]}{2\sqrt{2}(a^2+b^2)^3d} + \\
& \frac{(a-b)(a^2+4ab+b^2) e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e}\cot[c+dx] + \sqrt{2}\sqrt{e\cot[c+dx]}\right]}{2\sqrt{2}(a^2+b^2)^3d}
\end{aligned}$$

Result (type 3, 851 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \cot [c + d x])^3} (e \cot [c + d x])^{3/2} \operatorname{Csc}[c + d x]^2 \operatorname{Sec}[c + d x] (b \cos [c + d x] + a \sin [c + d x])^3 \\
& \left( \frac{a}{2 (-i a + b)^2 (i a + b)^2} - \frac{a b^2}{2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \frac{-7 a^2 \sin [c + d x] + 5 b^2 \sin [c + d x]}{4 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) + \\
& \frac{1}{8 (a - i b)^2 (a + i b)^2 d \cot [c + d x]^{3/2} (a + b \cot [c + d x])^3} (e \cot [c + d x])^{3/2} \operatorname{Csc}[c + d x]^3 \\
& (b \cos [c + d x] + a \sin [c + d x])^3 \left( - \frac{2 (-a^2 - b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^3 \operatorname{Sec}[c + d x]}{\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} - \right. \\
& \left( (4 a^2 - 4 b^2) \cos [2 (c + d x)] (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^3 \right. \\
& \left. \frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \\
& \left. (a + b) \left( \operatorname{Log} [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \operatorname{Log} [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \operatorname{Sec}[c + d x] \Big/ \\
& (2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) - \frac{1}{(a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} \\
& 2 a b (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} (-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}]) + \right. \\
& \left. (a - b) \left( \operatorname{Log} [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \operatorname{Log} [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \operatorname{Sec}[c + d x]^2 \sin [2 (c + d x)] \Big)
\end{aligned}$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{e \cot [c + d x]}}{(a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 463 leaves, 16 steps):

$$\frac{\sqrt{b} (15 a^4 - 18 a^2 b^2 - b^4) \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c + d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 a^{3/2} (a^2 + b^2)^3 d} + \frac{(a - b) (a^2 + 4 a b + b^2) \sqrt{e} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} -$$

$$\frac{(a - b) (a^2 + 4 a b + b^2) \sqrt{e} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{b \sqrt{e \cot [c + d x]}}{2 (a^2 + b^2) d (a + b \cot [c + d x])^2} +$$

$$\frac{b (7 a^2 - b^2) \sqrt{e \cot [c + d x]}}{4 a (a^2 + b^2)^2 d (a + b \cot [c + d x])} - \frac{(a + b) (a^2 - 4 a b + b^2) \sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] - \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d} +$$

$$\frac{(a + b) (a^2 - 4 a b + b^2) \sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] + \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d}$$

Result (type 3, 852 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \operatorname{Cot}[c + dx])^3} \sqrt{e \operatorname{Cot}[c + dx]} \operatorname{Csc}[c + dx]^3 (b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^3 \\
& \left( -\frac{b}{2 (-i a + b)^2 (i a + b)^2} + \frac{b^3}{2 (-i a + b)^2 (i a + b)^2 (b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^2} + \frac{11 a^2 b \operatorname{Sin}[c + dx] - b^3 \operatorname{Sin}[c + dx]}{4 a (-i a + b)^2 (i a + b)^2 (b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])} \right) + \\
& \frac{1}{8 a (a - i b)^2 (a + i b)^2 d \sqrt{\operatorname{Cot}[c + dx]} (a + b \operatorname{Cot}[c + dx])^3} \sqrt{e \operatorname{Cot}[c + dx]} \operatorname{Csc}[c + dx]^3 \\
& (b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^3 \left( -\frac{2 (a^2 b + b^3) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\operatorname{Cot}[c + dx]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b} (1 + \operatorname{Cot}[c + dx])^2 (b + a \operatorname{Tan}[c + dx])} + \right. \\
& \frac{1}{(a^2 + b^2) (-1 + \operatorname{Cot}[c + dx])^2 (1 + \operatorname{Cot}[c + dx])^2 (b + a \operatorname{Tan}[c + dx])} 4 a^2 b \operatorname{Cos}[2 (c + dx)] (a + b \operatorname{Cot}[c + dx]) \operatorname{Csc}[c + dx]^3 \\
& \left. \left( \frac{4 (a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\operatorname{Cot}[c + dx]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]}] - 2 (a - b) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]}]) + \right. \right. \\
& \left. \left. (a + b) \left( \operatorname{Log}[1 - \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]} + \operatorname{Cot}[c + dx]] - \operatorname{Log}[1 + \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]} + \operatorname{Cot}[c + dx]] \right) \right) \right) \operatorname{Sec}[c + dx] - \\
& \frac{1}{4 (a^2 + b^2) (1 + \operatorname{Cot}[c + dx])^2 (b + a \operatorname{Tan}[c + dx])} (4 a^3 - 4 a b^2) (a + b \operatorname{Cot}[c + dx]) \operatorname{Csc}[c + dx]^2 \\
& \left( -8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\operatorname{Cot}[c + dx]}}{\sqrt{a}}\right] + \sqrt{2} (-2 (a + b) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]}] + 2 (a + b) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]}]) + \right. \\
& \left. (a - b) \left( \operatorname{Log}[1 - \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]} + \operatorname{Cot}[c + dx]] - \operatorname{Log}[1 + \sqrt{2} \sqrt{\operatorname{Cot}[c + dx]} + \operatorname{Cot}[c + dx]] \right) \right) \right) \operatorname{Sec}[c + dx]^2 \operatorname{Sin}[2 (c + dx)] \Big)
\end{aligned}$$

**Problem 86: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{e \operatorname{Cot}[c + dx]} (a + b \operatorname{Cot}[c + dx])^3} dx$$

Optimal (type 3, 476 leaves, 16 steps):

$$\begin{aligned}
& - \frac{b^{3/2} (35 a^4 + 6 a^2 b^2 + 3 b^4) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{a} \sqrt{e}}\right]}{4 a^{5/2} (a^2 + b^2)^3 d \sqrt{e}} + \frac{(a+b) (a^2 - 4 a b + b^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d \sqrt{e}} - \\
& \frac{(a+b) (a^2 - 4 a b + b^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d \sqrt{e}} - \frac{b^2 \sqrt{e \operatorname{Cot}[c+dx]}}{2 a (a^2 + b^2) d e (a + b \operatorname{Cot}[c+dx])^2} - \\
& \frac{b^2 (11 a^2 + 3 b^2) \sqrt{e \operatorname{Cot}[c+dx]}}{4 a^2 (a^2 + b^2)^2 d e (a + b \operatorname{Cot}[c+dx])} + \frac{(a-b) (a^2 + 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d \sqrt{e}} - \\
& \frac{(a-b) (a^2 + 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d \sqrt{e}}
\end{aligned}$$

Result (type 3, 879 leaves):

$$\begin{aligned}
& \left( \text{Cot}[c + dx] \text{Csc}[c + dx]^3 (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])^3 \left( \frac{b^2}{2a(-ib + b)^2 (ia + b)^2} - \frac{b^4}{2a(-ib + b)^2 (ia + b)^2 (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])^2} - \right. \right. \\
& \quad \left. \left. \frac{3(5a^2 b^2 \text{Sin}[c + dx] + b^4 \text{Sin}[c + dx])}{4a^2(-ib + b)^2 (ia + b)^2 (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])} \right) \right) / \left( d \sqrt{e \text{Cot}[c + dx]} (a + b \text{Cot}[c + dx])^3 \right) - \\
& \frac{1}{8a^2 (a - ib)^2 (a + ib)^2 d \sqrt{e \text{Cot}[c + dx]} (a + b \text{Cot}[c + dx])^3} \sqrt{\text{Cot}[c + dx]} \text{Csc}[c + dx]^3 (b \text{Cos}[c + dx] + a \text{Sin}[c + dx])^3 \\
& \left( - \frac{2(-4a^4 - 7a^2 b^2 - 3b^4) \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{\text{Cot}[c + dx]}}{\sqrt{a}}\right] (a + b \text{Cot}[c + dx]) \text{Csc}[c + dx]^3 \text{Sec}[c + dx]}{\sqrt{a} \sqrt{b} (1 + \text{Cot}[c + dx])^2 (b + a \text{Tan}[c + dx])} - \right. \\
& \left( (4a^4 - 4a^2 b^2) \text{Cos}[2(c + dx)] (a + b \text{Cot}[c + dx]) \text{Csc}[c + dx]^3 \right. \\
& \left. \left( \frac{4(a^2 - b^2) \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{\text{Cot}[c + dx]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2(a - b) \text{ArcTan}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]}] - 2(a - b) \text{ArcTan}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]}] + \right. \right. \\
& \quad \left. \left. (a + b) \left( \text{Log}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] - \text{Log}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] \right) \right) \right) \text{Sec}[c + dx] \left. \right) / \\
& \frac{1}{(2(a^2 + b^2) (-1 + \text{Cot}[c + dx])^2 (1 + \text{Cot}[c + dx])^2 (b + a \text{Tan}[c + dx]))} - \frac{1}{(a^2 + b^2) (1 + \text{Cot}[c + dx])^2 (b + a \text{Tan}[c + dx])} \\
& 2a^3 b (a + b \text{Cot}[c + dx]) \text{Csc}[c + dx]^2 \\
& \left( -8\sqrt{a} \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{\text{Cot}[c + dx]}}{\sqrt{a}}\right] + \sqrt{2} (-2(a + b) \text{ArcTan}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]}] + 2(a + b) \text{ArcTan}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]}] + \right. \\
& \quad \left. (a - b) \left( \text{Log}[1 - \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] - \text{Log}[1 + \sqrt{2} \sqrt{\text{Cot}[c + dx]} + \text{Cot}[c + dx]] \right) \right) \right) \text{Sec}[c + dx]^2 \text{Sin}[2(c + dx)] \left. \right)
\end{aligned}$$



**Problem 87: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(e \cot [c + d x])^{3/2} (a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 529 leaves, 17 steps):

$$\frac{b^{5/2} (63 a^4 + 46 a^2 b^2 + 15 b^4) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c + d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 a^{7/2} (a^2 + b^2)^3 d e^{3/2}} - \frac{(a - b) (a^2 + 4 a b + b^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d e^{3/2}} +$$

$$\frac{(a - b) (a^2 + 4 a b + b^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d e^{3/2}} + \frac{8 a^4 + 31 a^2 b^2 + 15 b^4}{4 a^3 (a^2 + b^2)^2 d e \sqrt{e \cot [c + d x]}} - \frac{b^2}{2 a (a^2 + b^2) d e \sqrt{e \cot [c + d x]} (a + b \cot [c + d x])^2} -$$

$$\frac{b^2 (13 a^2 + 5 b^2)}{4 a^2 (a^2 + b^2)^2 d e \sqrt{e \cot [c + d x]} (a + b \cot [c + d x])} + \frac{(a + b) (a^2 - 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] - \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d e^{3/2}} -$$

$$\frac{(a + b) (a^2 - 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c + d x] + \sqrt{2} \sqrt{e \cot [c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d e^{3/2}}$$

Result (type 3, 894 leaves):

$$\begin{aligned}
& \left( \text{Cot}[c+dx]^2 \text{Csc}[c+dx]^3 (b \text{Cos}[c+dx] + a \text{Sin}[c+dx])^3 \left( -\frac{b^3}{2a^2(-ia+b)^2(ia+b)^2} + \right. \right. \\
& \quad \left. \frac{b^5}{2a^2(-ia+b)^2(ia+b)^2(b \text{Cos}[c+dx] + a \text{Sin}[c+dx])^2} + \frac{19a^2b^3 \text{Sin}[c+dx] + 7b^5 \text{Sin}[c+dx]}{4a^3(-ia+b)^2(ia+b)^2(b \text{Cos}[c+dx] + a \text{Sin}[c+dx])} + \frac{2 \text{Tan}[c+dx]}{a^3} \right) \Bigg) / \\
& \left( d (e \text{Cot}[c+dx])^{3/2} (a+b \text{Cot}[c+dx])^3 \right) - \frac{1}{8a^3(a-ib)^2(a+ib)^2 d (e \text{Cot}[c+dx])^{3/2} (a+b \text{Cot}[c+dx])^3} \\
& \text{Cot}[c+dx]^{3/2} \text{Csc}[c+dx]^3 (b \text{Cos}[c+dx] + a \text{Sin}[c+dx])^3 \\
& \left( -\frac{2(16a^4b + 31a^2b^3 + 15b^5) \text{ArcTan}\left[\frac{\sqrt{b}\sqrt{\text{Cot}[c+dx]}}{\sqrt{a}}\right] (a+b \text{Cot}[c+dx]) \text{Csc}[c+dx]^3 \text{Sec}[c+dx]}{\sqrt{a}\sqrt{b}(1+\text{Cot}[c+dx]^2)^2(b+a \text{Tan}[c+dx])} + \right. \\
& \quad \frac{1}{(a^2+b^2)(-1+\text{Cot}[c+dx]^2)(1+\text{Cot}[c+dx]^2)(b+a \text{Tan}[c+dx])} 4a^4b \text{Cos}[2(c+dx)] (a+b \text{Cot}[c+dx]) \text{Csc}[c+dx]^3 \\
& \quad \left. \left( \frac{4(a^2-b^2) \text{ArcTan}\left[\frac{\sqrt{b}\sqrt{\text{Cot}[c+dx]}}{\sqrt{a}}\right]}{\sqrt{a}\sqrt{b}} + \sqrt{2} (2(a-b) \text{ArcTan}[1-\sqrt{2}\sqrt{\text{Cot}[c+dx]}] - 2(a-b) \text{ArcTan}[1+\sqrt{2}\sqrt{\text{Cot}[c+dx]}]) + \right. \right. \\
& \quad \left. \left. (a+b) \left( \text{Log}[1-\sqrt{2}\sqrt{\text{Cot}[c+dx]} + \text{Cot}[c+dx]] - \text{Log}[1+\sqrt{2}\sqrt{\text{Cot}[c+dx]} + \text{Cot}[c+dx]] \right) \right) \right) \text{Sec}[c+dx] - \\
& \frac{1}{4(a^2+b^2)(1+\text{Cot}[c+dx]^2)(b+a \text{Tan}[c+dx])} (4a^5 - 4a^3b^2) (a+b \text{Cot}[c+dx]) \text{Csc}[c+dx]^2 \\
& \left( -8\sqrt{a}\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b}\sqrt{\text{Cot}[c+dx]}}{\sqrt{a}}\right] + \sqrt{2} (-2(a+b) \text{ArcTan}[1-\sqrt{2}\sqrt{\text{Cot}[c+dx]}] + 2(a+b) \text{ArcTan}[1+\sqrt{2}\sqrt{\text{Cot}[c+dx]}]) + \right. \\
& \quad \left. (a-b) \left( \text{Log}[1-\sqrt{2}\sqrt{\text{Cot}[c+dx]} + \text{Cot}[c+dx]] - \text{Log}[1+\sqrt{2}\sqrt{\text{Cot}[c+dx]} + \text{Cot}[c+dx]] \right) \right) \right) \text{Sec}[c+dx]^2 \text{Sin}[2(c+dx)] \Bigg)
\end{aligned}$$

**Problem 88: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \text{Cot}[c+dx])^n dx$$

Optimal (type 5, 167 leaves, 5 steps):

$$-\frac{b (a + b \operatorname{Cot}[c + d x])^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{a + b \operatorname{Cot}[c + d x]}{a - \sqrt{-b^2}}\right]}{2 \sqrt{-b^2} (a - \sqrt{-b^2}) d (1 + n)} + \frac{b (a + b \operatorname{Cot}[c + d x])^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{a + b \operatorname{Cot}[c + d x]}{a + \sqrt{-b^2}}\right]}{2 \sqrt{-b^2} (a + \sqrt{-b^2}) d (1 + n)}$$

Result (type 5, 161 leaves):

$$\frac{1}{2 d n} i (a + b \operatorname{Cot}[c + d x])^n \left( \left( \frac{a + b \operatorname{Cot}[c + d x]}{b (-i + \operatorname{Cot}[c + d x])} \right)^{-n} \operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{a + i b}{b (-i + \operatorname{Cot}[c + d x])}\right] - \left( \frac{a + b \operatorname{Cot}[c + d x]}{b (i + \operatorname{Cot}[c + d x])} \right)^{-n} \operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, \frac{-a + i b}{b (i + \operatorname{Cot}[c + d x])}\right] \right)$$

Problem 89: Unable to integrate problem.

$$\int (a + b \operatorname{Cot}[e + f x])^m (d \operatorname{Tan}[e + f x])^n dx$$

Optimal (type 6, 193 leaves, 8 steps):

$$-\frac{1}{2 f (1 - n)} \operatorname{AppellF1}\left[1 - n, -m, 1, 2 - n, -\frac{b \operatorname{Cot}[e + f x]}{a}, -i \operatorname{Cot}[e + f x]\right] \operatorname{Cot}[e + f x] (a + b \operatorname{Cot}[e + f x])^m \left(1 + \frac{b \operatorname{Cot}[e + f x]}{a}\right)^{-m} (d \operatorname{Tan}[e + f x])^n - \frac{1}{2 f (1 - n)} \operatorname{AppellF1}\left[1 - n, -m, 1, 2 - n, -\frac{b \operatorname{Cot}[e + f x]}{a}, i \operatorname{Cot}[e + f x]\right] \operatorname{Cot}[e + f x] (a + b \operatorname{Cot}[e + f x])^m \left(1 + \frac{b \operatorname{Cot}[e + f x]}{a}\right)^{-m} (d \operatorname{Tan}[e + f x])^n$$

Result (type 8, 25 leaves):

$$\int (a + b \operatorname{Cot}[e + f x])^m (d \operatorname{Tan}[e + f x])^n dx$$

Problem 91: Result more than twice size of optimal antiderivative.

$$\int \frac{1 - i \operatorname{Cot}[c + d x]}{\sqrt{a + b \operatorname{Cot}[c + d x]}} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$-\frac{2 i \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Cot}[c + d x]}}{\sqrt{a + i b}}\right]}{\sqrt{a + i b} d}$$

Result (type 3, 128 leaves):

$$\frac{i \operatorname{Log} \left[ \frac{2 \left( i b e^{2i(c+dx)} + a (-1 + e^{2i(c+dx)}) + \sqrt{a+ib} (-1 + e^{2i(c+dx)}) \sqrt{a + \frac{ib(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}} \right)}{\sqrt{a+ib}} \right]}{\sqrt{a+ib} d}$$

**Problem 93: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cot}[c + dx]}{(a + b \operatorname{Cot}[c + dx])^2} dx$$

Optimal (type 3, 111 leaves, 3 steps):

$$\frac{(a^2 A - A b^2 + 2 a b B) x}{(a^2 + b^2)^2} + \frac{A b - a B}{(a^2 + b^2) d (a + b \operatorname{Cot}[c + dx])} - \frac{(2 a A b - a^2 B + b^2 B) \operatorname{Log}[b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx]]}{(a^2 + b^2)^2 d}$$

Result (type 3, 352 leaves):

$$\frac{1}{2 (a^2 + b^2)^2 d (a + b \operatorname{Cot}[c + dx])} \left( 2 a^2 A b + 2 A b^3 - 2 a^3 B - 2 a b^2 B + 2 a^3 A c - 4 i a^2 A b c - 2 a A b^2 c + 2 i a^3 B c + 4 a^2 b B c - 2 i a b^2 B c + 2 a^3 A d x - 4 i a^2 A b d x - 2 a A b^2 d x + 2 i a^3 B d x + 4 a^2 b B d x - 2 i a b^2 B d x - 2 i (-2 a A b + a^2 B - b^2 B) \operatorname{ArcTan}[\operatorname{Tan}[c + dx]] (a + b \operatorname{Cot}[c + dx]) - 2 a^2 A b \operatorname{Log}[(b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^2] + a^3 B \operatorname{Log}[(b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^2] - a b^2 B \operatorname{Log}[(b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^2] + b \operatorname{Cot}[c + dx] (2 (a - i b)^2 (A + i B) (c + dx) + (-2 a A b + a^2 B - b^2 B) \operatorname{Log}[(b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx])^2]) \right)$$

**Problem 94: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Cot}[c + dx]}{(a + b \operatorname{Cot}[c + dx])^3} dx$$

Optimal (type 3, 175 leaves, 4 steps):

$$\frac{(a^3 A - 3 a A b^2 + 3 a^2 b B - b^3 B) x}{(a^2 + b^2)^3} + \frac{A b - a B}{2 (a^2 + b^2) d (a + b \operatorname{Cot}[c + dx])^2} + \frac{2 a A b - a^2 B + b^2 B}{(a^2 + b^2)^2 d (a + b \operatorname{Cot}[c + dx])} - \frac{(3 a^2 A b - A b^3 - a^3 B + 3 a b^2 B) \operatorname{Log}[b \operatorname{Cos}[c + dx] + a \operatorname{Sin}[c + dx]]}{(a^2 + b^2)^3 d}$$

Result (type 3, 863 leaves):

$$\begin{aligned}
& \frac{b^2 (A b - a B) (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])}{2 (-i a + b)^2 (i a + b)^2 d (a + b \cot [c + d x])^3 (B \cos [c + d x] + A \sin [c + d x])} - \\
& \left( (-a^3 A + 3 a A b^2 - 3 a^2 b B + b^3 B) (c + d x) (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right) / \\
& \left( (-i a + b)^3 (i a + b)^3 d (a + b \cot [c + d x])^3 (B \cos [c + d x] + A \sin [c + d x]) \right) + \\
& \left( (-3 i a^7 A b^3 + 3 a^6 A b^4 - 5 i a^5 A b^5 + 5 a^4 A b^6 - i a^3 A b^7 + a^2 A b^8 + i a A b^9 - A b^{10} + i a^8 b^2 B - a^7 b^3 B - i a^6 b^4 B + a^5 b^5 B - \right. \\
& \quad \left. 5 i a^4 b^6 B + 5 a^3 b^7 B - 3 i a^2 b^8 B + 3 a b^9 B) (c + d x) (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right) / \\
& \left( (a - i b)^2 (a + i b)^3 b^2 (-i a + b)^3 (i a + b)^3 d (a + b \cot [c + d x])^3 (B \cos [c + d x] + A \sin [c + d x]) \right) - \\
& \left( i (-3 a^2 A b + A b^3 + a^3 B - 3 a b^2 B) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right) / \\
& \left( (a^2 + b^2)^3 d (a + b \cot [c + d x])^3 (B \cos [c + d x] + A \sin [c + d x]) \right) + \\
& \left( (-3 a^2 A b + A b^3 + a^3 B - 3 a b^2 B) (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 \operatorname{Log}[(b \cos [c + d x] + a \sin [c + d x])^2 (b \cos [c + d x] + a \sin [c + d x])^3] \right) / \\
& \left( 2 (a^2 + b^2)^3 d (a + b \cot [c + d x])^3 (B \cos [c + d x] + A \sin [c + d x]) \right) + \\
& \left( (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^2 (3 a A b \sin [c + d x] - 2 a^2 B \sin [c + d x] + b^2 B \sin [c + d x]) \right) / \\
& \left( (-i a + b)^2 (i a + b)^2 d (a + b \cot [c + d x])^3 (B \cos [c + d x] + A \sin [c + d x]) \right)
\end{aligned}$$

**Problem 95: Result more than twice size of optimal antiderivative.**

$$\int (a + b \cot [c + d x])^{5/2} (A + B \cot [c + d x]) dx$$

Optimal (type 3, 188 leaves, 10 steps):

$$\begin{aligned}
& \frac{(a - i b)^{5/2} (i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}}\right]}{d} - \frac{(a + i b)^{5/2} (i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}}\right]}{d} - \\
& \frac{2 (2 a A b + a^2 B - b^2 B) \sqrt{a + b \cot [c + d x]}}{d} - \frac{2 (A b + a B) (a + b \cot [c + d x])^{3/2}}{3 d} - \frac{2 B (a + b \cot [c + d x])^{5/2}}{5 d}
\end{aligned}$$

Result (type 3, 505 leaves):

$$\begin{aligned}
& \left( i (a^3 A - 3 a A b^2 - 3 a^2 b B + b^3 B) \left( \frac{\text{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} - \frac{\text{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) (a+b \cot [c+d x])^3 (A+B \cot [c+d x]) \sin [c+d x]^4 \right) / \\
& \left( d (b \cos [c+d x] + a \sin [c+d x])^3 (B \cos [c+d x] + A \sin [c+d x]) \right) + \\
& \left( (3 a^2 A b - A b^3 + a^3 B - 3 a b^2 B) \left( \frac{\text{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} + \frac{\text{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) (a+b \cot [c+d x])^3 (A+B \cot [c+d x]) \sin [c+d x]^4 \right) / \\
& \left( d (b \cos [c+d x] + a \sin [c+d x])^3 (B \cos [c+d x] + A \sin [c+d x]) \right) + \left( (a+b \cot [c+d x])^{5/2} (A+B \cot [c+d x]) \right. \\
& \left. \left( \frac{2}{15} (-35 a A b - 23 a^2 B + 18 b^2 B) - \frac{2}{15} (5 A b^2 \cos [c+d x] + 11 a b B \cos [c+d x]) \csc [c+d x] - \frac{2}{5} b^2 B \csc [c+d x]^2 \right) \sin [c+d x]^3 \right) / \\
& \left( d (b \cos [c+d x] + a \sin [c+d x])^2 (B \cos [c+d x] + A \sin [c+d x]) \right)
\end{aligned}$$

**Problem 96: Result more than twice size of optimal antiderivative.**

$$\int (a+b \cot [c+d x])^{3/2} (A+B \cot [c+d x]) dx$$

Optimal (type 3, 150 leaves, 9 steps):

$$\begin{aligned}
& \frac{(a-i b)^{3/2} (i A+B) \text{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{d} - \\
& \frac{(a+i b)^{3/2} (i A-B) \text{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{d} - \frac{2 (A b+a B) \sqrt{a+b \cot [c+d x]}}{d} - \frac{2 B (a+b \cot [c+d x])^{3/2}}{3 d}
\end{aligned}$$

Result (type 3, 441 leaves):

$$\begin{aligned}
& \left( i (a^2 A - A b^2 - 2 a b B) \left( \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) (a+b \cot [c+d x])^2 (A+B \cot [c+d x]) \sin [c+d x]^3 \right) / \\
& \left( d (b \cos [c+d x] + a \sin [c+d x])^2 (B \cos [c+d x] + A \sin [c+d x]) \right) + \\
& \left( (2 a A b + a^2 B - b^2 B) \left( \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) (a+b \cot [c+d x])^2 (A+B \cot [c+d x]) \sin [c+d x]^3 \right) / \\
& \left( d (b \cos [c+d x] + a \sin [c+d x])^2 (B \cos [c+d x] + A \sin [c+d x]) \right) + \\
& \frac{(a+b \cot [c+d x])^{3/2} (A+B \cot [c+d x]) \left( -\frac{2}{3} (3 A b + 4 a B) - \frac{2}{3} b B \cot [c+d x] \right) \sin [c+d x]^2}{d (b \cos [c+d x] + a \sin [c+d x]) (B \cos [c+d x] + A \sin [c+d x])}
\end{aligned}$$

**Problem 98: Result more than twice size of optimal antiderivative.**

$$\int (-a + b \cot [c + d x]) (a + b \cot [c + d x])^{5/2} dx$$

Optimal (type 3, 151 leaves, 10 steps):

$$\begin{aligned}
& -\frac{(i a - b) (a - i b)^{5/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{d} + \\
& \frac{(a + i b)^{5/2} (i a + b) \operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{d} + \frac{2 b (a^2 + b^2) \sqrt{a+b \cot [c+d x]}}{d} - \frac{2 b (a + b \cot [c+d x])^{5/2}}{5 d}
\end{aligned}$$

Result (type 3, 479 leaves):

$$\left( (-a + b \cot[c + dx]) (a + b \cot[c + dx])^{5/2} \left( -\frac{4}{5} b (2a^2 + 3b^2) + \frac{4}{5} a b^2 \cot[c + dx] + \frac{2}{5} b^3 \csc[c + dx]^2 \right) \sin[c + dx]^3 \right) /$$

$$\left( d (-b \cos[c + dx] + a \sin[c + dx]) (b \cos[c + dx] + a \sin[c + dx])^2 \right) +$$

$$\left( (a^2 + b^2) (-a + b \cot[c + dx]) (a + b \cot[c + dx])^{5/2} \frac{i (a^2 - b^2) \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a-b \cot[c+dx]}}{\sqrt{a-ib}}\right]}{\sqrt{a-ib}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a-b \cot[c+dx]}}{\sqrt{a+ib}}\right]}{\sqrt{a+ib}} \right) \sqrt{a + b \cot[c + dx]}}{\sqrt{\csc[c + dx]} \sqrt{b \cos[c + dx] + a \sin[c + dx]}} + \right.$$

$$\left. \frac{2 a b \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a-b \cot[c+dx]}}{\sqrt{a-ib}}\right]}{\sqrt{a-ib}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a-b \cot[c+dx]}}{\sqrt{a+ib}}\right]}{\sqrt{a+ib}} \right) \sqrt{a + b \cot[c + dx]}}{\sqrt{\csc[c + dx]} \sqrt{b \cos[c + dx] + a \sin[c + dx]}} \right) /$$

$$\left( d \csc[c + dx]^{7/2} (-b \cos[c + dx] + a \sin[c + dx]) (b \cos[c + dx] + a \sin[c + dx])^{5/2} \right)$$

**Problem 99: Result unnecessarily involves imaginary or complex numbers.**

$$\int (-a + b \cot[c + dx]) (a + b \cot[c + dx])^{3/2} dx$$

Optimal (type 3, 408 leaves, 13 steps):

$$\frac{b (a^2 + b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+\sqrt{a^2+b^2}} - \sqrt{2} \sqrt{a+b \cot[c+dx]}}{\sqrt{a-\sqrt{a^2+b^2}}}\right]}{\sqrt{2} \sqrt{a-\sqrt{a^2+b^2}} d} - \frac{b (a^2 + b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+\sqrt{a^2+b^2}} + \sqrt{2} \sqrt{a+b \cot[c+dx]}}{\sqrt{a-\sqrt{a^2+b^2}}}\right]}{\sqrt{2} \sqrt{a-\sqrt{a^2+b^2}} d} -$$

$$\frac{2 b (a + b \cot[c + dx])^{3/2}}{3 d} + \frac{b (a^2 + b^2) \operatorname{Log}\left[\frac{a + \sqrt{a^2 + b^2} + b \cot[c + dx] - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot[c + dx]}}{2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d}\right]}{2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d} -$$

$$\frac{b (a^2 + b^2) \operatorname{Log}\left[\frac{a + \sqrt{a^2 + b^2} + b \cot[c + dx] + \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot[c + dx]}}{2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d}\right]}{2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d}$$



Result (type 3, 178 leaves):

$$\left( (-a + b \cot [c + d x]) (a + b \cot [c + d x]) \right. \\ \left. \left( 3 i \sqrt{a - i b} (a^2 + b^2) \operatorname{ArcTanh} \left[ \frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}} \right] - 3 i \sqrt{a + i b} (a^2 + b^2) \operatorname{ArcTanh} \left[ \frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}} \right] + 2 b (a + b \cot [c + d x])^{3/2} \right) \right. \\ \left. \sin [c + d x]^2 \right) / \left( -3 b^2 d \cos [c + d x]^2 + 3 a^2 d \sin [c + d x]^2 \right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int (-a + b \cot [c + d x]) \sqrt{a + b \cot [c + d x]} dx$$

Optimal (type 3, 422 leaves, 13 steps):

$$\frac{b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \cot [c + d x]}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right] - b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \cot [c + d x]}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right]}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{2 b \sqrt{a + b \cot [c + d x]}}{d} - \frac{b \sqrt{a^2 + b^2} \operatorname{Log} [a + \sqrt{a^2 + b^2} + b \cot [c + d x] - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot [c + d x]}]}{2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d} + \frac{b \sqrt{a^2 + b^2} \operatorname{Log} [a + \sqrt{a^2 + b^2} + b \cot [c + d x] + \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot [c + d x]}]}{2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d}$$

Result (type 3, 158 leaves):

$$\left( (-a + b \cot [c + d x]) \left( \frac{i (a^2 + b^2) \operatorname{ArcTanh} \left[ \frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}} \right]}{\sqrt{a - i b}} - \frac{i (a^2 + b^2) \operatorname{ArcTanh} \left[ \frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}} \right]}{\sqrt{a + i b}} + 2 b \sqrt{a + b \cot [c + d x]} \right) \sin [c + d x] \right) / \left( d (-b \cos [c + d x] + a \sin [c + d x]) \right)$$

### Problem 102: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cot [c + d x]}{(a + b \cot [c + d x])^{3/2}} dx$$

Optimal (type 3, 138 leaves, 8 steps):

$$\frac{(i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{(a-i b)^{3/2} d} - \frac{(i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{(a+i b)^{3/2} d} + \frac{2(A b - a B)}{(a^2 + b^2) d \sqrt{a + b \cot [c + d x]}}$$

Result (type 3, 476 leaves):

$$\frac{2(A + B \cot [c + d x]) \operatorname{Csc}[c + d x] (b \cos [c + d x] + a \sin [c + d x]) (A b \sin [c + d x] - a B \sin [c + d x])}{(-i a + b) (i a + b) d (a + b \cot [c + d x])^{3/2} (B \cos [c + d x] + A \sin [c + d x])} +$$

$$\left( \frac{(A + B \cot [c + d x]) \sqrt{\operatorname{Csc}[c + d x]} (b \cos [c + d x] + a \sin [c + d x])^{3/2}}{\sqrt{\operatorname{Csc}[c + d x]} \sqrt{b \cos [c + d x] + a \sin [c + d x]}} \left( \frac{i (a A + b B) \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \sqrt{a + b \cot [c + d x]}}{\sqrt{\operatorname{Csc}[c + d x]} \sqrt{b \cos [c + d x] + a \sin [c + d x]}} \right) + \right.$$

$$\left. \frac{(-A b + a B) \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \sqrt{a + b \cot [c + d x]}}{\sqrt{\operatorname{Csc}[c + d x]} \sqrt{b \cos [c + d x] + a \sin [c + d x]}} \right) /$$

$$\left( (a - i b) (a + i b) d (a + b \cot [c + d x])^{3/2} (B \cos [c + d x] + A \sin [c + d x]) \right)$$

### Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cot [c + d x]}{(a + b \cot [c + d x])^{5/2}} dx$$

Optimal (type 3, 185 leaves, 9 steps):

$$\frac{(i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{(a-i b)^{5/2} d} - \frac{(i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{(a+i b)^{5/2} d} + \frac{2(A b - a B)}{3(a^2 + b^2) d (a+b \cot [c+d x])^{3/2}} + \frac{2(2 a A b - a^2 B + b^2 B)}{(a^2 + b^2)^2 d \sqrt{a+b \cot [c+d x]}}$$

Result (type 3, 620 leaves):

$$\left( (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^{3/2} (b \cos [c + d x] + a \sin [c + d x])^{5/2} \right.$$

$$\left. \frac{(i(a^2 A - A b^2 + 2 a b B) \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]}}{\sqrt{\operatorname{Csc}[c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]}} + \right.$$

$$\left. \frac{(-2 a A b + a^2 B - b^2 B) \left( \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]}}{\sqrt{\operatorname{Csc}[c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]}} \right) \left. \right) / \left( (a-i b)^2 (a+i b)^2 d (a+b \cot [c+d x])^{5/2} (B \cos [c+d x] + A \sin [c+d x]) \right) +$$

$$\left( (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right.$$

$$\left. - \frac{2(A b - a B)}{3(-i a + b)^2 (i a + b)^2} + \frac{2 b^2 (A b - a B)}{3(-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \right.$$

$$\left. \frac{2(8 a A b \sin [c + d x] - 5 a^2 B \sin [c + d x] + 3 b^2 B \sin [c + d x])}{3(-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) / \left( d (a+b \cot [c+d x])^{5/2} (B \cos [c+d x] + A \sin [c+d x]) \right)$$

### Problem 106: Result more than twice size of optimal antiderivative.

$$\int \frac{-a + b \operatorname{Cot}[c + d x]}{(a + b \operatorname{Cot}[c + d x])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 9 steps):

$$-\frac{(i a - b) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Cot}[c + d x]}}{\sqrt{a - i b}}\right]}{(a - i b)^{5/2} d} + \frac{(i a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Cot}[c + d x]}}{\sqrt{a + i b}}\right]}{(a + i b)^{5/2} d} - \frac{4 a b}{3 (a^2 + b^2) d (a + b \operatorname{Cot}[c + d x])^{3/2}} - \frac{2 b (3 a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \operatorname{Cot}[c + d x]}}$$

Result (type 3, 587 leaves):

$$\left( (-a + b \cot [c + d x]) \operatorname{Csc} [c + d x]^{3/2} \right. \\
\left. (b \cos [c + d x] + a \sin [c + d x])^{5/2} \left( \frac{i (a^3 - 3 a b^2) \left( \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]}}{\sqrt{\operatorname{Csc} [c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]}} + \right. \right. \\
\left. \left. \frac{(-3 a^2 b + b^3) \left( \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]}}{\sqrt{\operatorname{Csc} [c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]}} \right) \right) / \\
\left( (a - i b)^2 (a + i b)^2 d (a + b \cot [c + d x])^{5/2} (-b \cos [c + d x] + a \sin [c + d x]) \right) + \\
\left( (-a + b \cot [c + d x]) \operatorname{Csc} [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right. \\
\left. \left( -\frac{4 a b}{3 (-i a + b)^2 (i a + b)^2} + \frac{4 a b^3}{3 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} - \right. \right. \\
\left. \left. \frac{2 (-13 a^2 b \sin [c + d x] + 3 b^3 \sin [c + d x])}{3 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) \right) / \left( d (a + b \cot [c + d x])^{5/2} (-b \cos [c + d x] + a \sin [c + d x]) \right)$$

Test results for the 64 problems in "4.4.7 (d trig)^m (a+b (c cot)^n)^p.m"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int (1 + \cot [x]^2)^{3/2} dx$$

Optimal (type 3, 22 leaves, 4 steps):

$$-\frac{1}{2} \operatorname{ArcSinh}[\operatorname{Cot}[x]] - \frac{1}{2} \operatorname{Cot}[x] \sqrt{\operatorname{Csc}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8} \sqrt{\operatorname{Csc}[x]^2} \left( -\operatorname{Csc}\left[\frac{x}{2}\right]^2 - 4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] + 4 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Sec}\left[\frac{x}{2}\right]^2 \right) \operatorname{Sin}[x]$$

**Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{1 + \operatorname{Cot}[x]^2} \, dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$-\operatorname{ArcSinh}[\operatorname{Cot}[x]]$$

Result (type 3, 28 leaves):

$$\sqrt{\operatorname{Csc}[x]^2} \left( -\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] \right) \operatorname{Sin}[x]$$

**Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{-1 - \operatorname{Cot}[x]^2} \, dx$$

Optimal (type 3, 14 leaves, 4 steps):

$$\operatorname{ArcTan}\left[\frac{\operatorname{Cot}[x]}{\sqrt{-\operatorname{Csc}[x]^2}}\right]$$

Result (type 3, 30 leaves):

$$\frac{\operatorname{Csc}[x] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] \right)}{\sqrt{-\operatorname{Csc}[x]^2}}$$

**Problem 19: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[x]^3 \sqrt{a + b \operatorname{Cot}[x]^2} \, dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$-\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [x]^2}}{\sqrt{a-b}}\right] + \sqrt{a+b \cot [x]^2} - \frac{(a+b \cot [x]^2)^{3/2}}{3b}$$

Result (type 4, 505 leaves):

$$\begin{aligned} & \sqrt{\frac{-a-b+a \cos [2x]-b \cos [2x]}{-1+\cos [2x]}} \left( \frac{-a+4b}{3b} - \frac{\operatorname{Csc}[x]^2}{3} \right) + \\ & \left( 2i(a-b)(1+\cos [x]) \sqrt{\frac{-1+\cos [2x]}{(1+\cos [x])^2}} \sqrt{\frac{-a-b+(a-b) \cos [2x]}{-1+\cos [2x]}} \left( \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right] \right], \right. \right. \right. \\ & \left. \left. \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b} \right] - 2 \operatorname{EllipticPi}\left[ \frac{2a+2\sqrt{a(a-b)}-b}{b}, i \operatorname{ArcSinh}\left[ \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right] \right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b} \right] \right) \\ & \tan\left[\frac{x}{2}\right] \sqrt{1+\frac{b \tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}} \sqrt{1-\frac{b \tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \Big/ \\ & \left( \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \sqrt{-a-b+(a-b) \cos [2x]} \sqrt{-\tan\left[\frac{x}{2}\right]^2} \left(1+\tan\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4a \tan\left[\frac{x}{2}\right]^2+b(-1+\tan\left[\frac{x}{2}\right]^2)^2}{(1+\tan\left[\frac{x}{2}\right]^2)^2}} \right) \end{aligned}$$

**Problem 20:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cot [x] \sqrt{a+b \cot [x]^2} dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [x]^2}}{\sqrt{a-b}}\right] - \sqrt{a+b \cot [x]^2}$$

Result (type 4, 363 leaves):

$$\frac{1}{\sqrt{2}} \sqrt{-(-a-b + (a-b) \cos[2x]) \csc[x]^2}$$

$$\left( -1 + \left( 8i (a-b) \cos\left[\frac{x}{2}\right]^3 \left( \text{EllipticF}\left[ i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right] \right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b} \right] - \right. \right.$$

$$\left. \left. 2 \text{EllipticPi}\left[ \frac{2a+2\sqrt{a(a-b)}-b}{b}, i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right] \right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b} \right] \right) \sin\left[\frac{x}{2}\right] \right.$$

$$\left. \sqrt{1 + \frac{b \tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}} \sqrt{1 - \frac{b \tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \right) / \left( \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} (a+b + (-a+b) \cos[2x]) \right)$$

**Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \cot[x]^2} \tan[x] \, dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a}}\right] - \sqrt{a-b} \text{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a-b}}\right]$$

Result (type 3, 197 leaves):

$$\frac{1}{2\sqrt{a-b} \sqrt{b+a \tan[x]^2}} \sqrt{a+b \cot[x]^2} \left( 2\sqrt{a} \sqrt{a-b} \text{Log}[a \tan[x] + \sqrt{a} \sqrt{b+a \tan[x]^2}] + \right.$$

$$\left. (a-b) \left( \text{Log}\left[\frac{4(b+i a \tan[x] - i \sqrt{a-b} \sqrt{b+a \tan[x]^2})}{(a-b)^{3/2} (-i + \tan[x])}\right] - \text{Log}\left[\frac{4i(i b + a \tan[x] + \sqrt{a-b} \sqrt{b+a \tan[x]^2})}{(a-b)^{3/2} (i + \tan[x])}\right] \right) \right) \tan[x]$$

**Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \cot[x]^2 \sqrt{a+b \cot[x]^2} \, dx$$

Optimal (type 3, 89 leaves, 7 steps):



$$\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \cot[x]}{\sqrt{a+b \cot[x]^2}}\right] - \frac{(a-2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cot[x]}{\sqrt{a+b \cot[x]^2}}\right]}{2\sqrt{b}} - \frac{1}{2} \cot[x] \sqrt{a+b \cot[x]^2}$$

Result (type 3, 2937 leaves):

$$-\frac{1}{2} \sqrt{\frac{-a-b+a \cos[2x]-b \cos[2x]}{-1+\cos[2x]}} \cot[x] +$$

$$\left( \frac{b \sqrt{\frac{-\frac{a}{-1+\cos[2x]} - \frac{b}{-1+\cos[2x]} + \frac{a \cos[2x]}{-1+\cos[2x]} - \frac{b \cos[2x]}{-1+\cos[2x]}}{-a-b+a \cos[2x]-b \cos[2x]}} - \frac{a \cos[2x] \sqrt{\frac{-\frac{a}{-1+\cos[2x]} - \frac{b}{-1+\cos[2x]} + \frac{a \cos[2x]}{-1+\cos[2x]} - \frac{b \cos[2x]}{-1+\cos[2x]}}{-a-b+a \cos[2x]-b \cos[2x]}}}{-a-b+a \cos[2x]-b \cos[2x]} + \right.$$

$$\left. \frac{b \cos[2x] \sqrt{\frac{-\frac{a}{-1+\cos[2x]} - \frac{b}{-1+\cos[2x]} + \frac{a \cos[2x]}{-1+\cos[2x]} - \frac{b \cos[2x]}{-1+\cos[2x]}}{-a-b+a \cos[2x]-b \cos[2x]}}}{-a-b+a \cos[2x]-b \cos[2x]} \right) \sqrt{a+b \cot[x]^2}$$

$$\left( 4\sqrt{b} \sqrt{-a+b} \operatorname{Log}\left[\operatorname{Sec}\left[\frac{x}{2}\right]^2\right] + (a-2b) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{x}{2}\right]^2\right] - a \operatorname{Log}\left[b + (2a-b) \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + \sqrt{b} \sqrt{b \cos[x]^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 + 4a \operatorname{Tan}\left[\frac{x}{2}\right]^2}\right] + \right.$$

$$2b \operatorname{Log}\left[b + (2a-b) \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + \sqrt{b} \sqrt{b \cos[x]^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 + 4a \operatorname{Tan}\left[\frac{x}{2}\right]^2}\right] +$$

$$a \operatorname{Log}\left[2a-b + b \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + \sqrt{b} \sqrt{b \cos[x]^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 + 4a \operatorname{Tan}\left[\frac{x}{2}\right]^2}\right] - 2b \operatorname{Log}\left[2a-b + b \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + \sqrt{b} \sqrt{b \cos[x]^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 + 4a \operatorname{Tan}\left[\frac{x}{2}\right]^2}\right] -$$

$$4\sqrt{b} \sqrt{-a+b} \operatorname{Log}\left[-a+b + (a-b) \operatorname{Tan}\left[\frac{x}{2}\right]^2\right] + \sqrt{-a+b} \sqrt{b \cos[x]^2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 + 4a \operatorname{Tan}\left[\frac{x}{2}\right]^2}\right) \operatorname{Tan}\left[\frac{x}{2}\right] \Big/$$

$$\left( \sqrt{2} \sqrt{b} \sqrt{(a+b+(-a+b) \cos[2x]) \operatorname{Sec}\left[\frac{x}{2}\right]^4} \frac{1}{2\sqrt{2} \sqrt{b} \sqrt{(a+b+(-a+b) \cos[2x]) \operatorname{Sec}\left[\frac{x}{2}\right]^4}} \sqrt{a+b \cot[x]^2} \right.$$



$$\frac{1}{\sqrt{2} \sqrt{b} \sqrt{(a+b+(-a+b) \cos[2x]) \sec\left[\frac{x}{2}\right]^4}} \sqrt{a+b \cot[x]^2 \tan\left[\frac{x}{2}\right]} \left( (a-2b) \csc\left[\frac{x}{2}\right] \sec\left[\frac{x}{2}\right] + 4\sqrt{b} \sqrt{-a+b} \tan\left[\frac{x}{2}\right] - \right.$$

$$\left. \frac{a \left( (2a-b) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + \frac{\sqrt{b} \left( -2b \cos[x] \sec\left[\frac{x}{2}\right]^4 \sin[x] + 4a \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + 2b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right] \right)}{2 \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} \right)}{b + (2a-b) \tan\left[\frac{x}{2}\right]^2 + \sqrt{b} \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} + \right.$$

$$\left. \left( 2b \left( (2a-b) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + \frac{\sqrt{b} \left( -2b \cos[x] \sec\left[\frac{x}{2}\right]^4 \sin[x] + 4a \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + 2b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right] \right)}{2 \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} \right) \right) \right) /$$

$$\left( b + (2a-b) \tan\left[\frac{x}{2}\right]^2 + \sqrt{b} \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2} \right) +$$

$$\left. \frac{a \left( b \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + \frac{\sqrt{b} \left( -2b \cos[x] \sec\left[\frac{x}{2}\right]^4 \sin[x] + 4a \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + 2b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right] \right)}{2 \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} \right)}{2a - b + b \tan\left[\frac{x}{2}\right]^2 + \sqrt{b} \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} - \right.$$

$$\left. \frac{2b \left( b \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + \frac{\sqrt{b} \left( -2b \cos[x] \sec\left[\frac{x}{2}\right]^4 \sin[x] + 4a \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + 2b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right] \right)}{2 \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} \right)}{2a - b + b \tan\left[\frac{x}{2}\right]^2 + \sqrt{b} \sqrt{b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 + 4a \tan\left[\frac{x}{2}\right]^2}} - \left( 4\sqrt{b} \sqrt{-a+b} \right) \right.$$

$$\left. \left( (a-b) \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + \left( \sqrt{-a+b} \left( -2b \cos[x] \sec\left[\frac{x}{2}\right]^4 \sin[x] + 4a \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] + 2b \cos[x]^2 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right] \right) \right) \right) /$$

$$\left( 2 \sqrt{b \cos^2 \left[ \frac{x}{2} \right] \sec^4 \left[ \frac{x}{2} \right] + 4 a \tan^2 \left[ \frac{x}{2} \right]} \right) / \left( -a + b + (a - b) \tan^2 \left[ \frac{x}{2} \right] + \sqrt{-a + b} \sqrt{b \cos^2 \left[ \frac{x}{2} \right] \sec^4 \left[ \frac{x}{2} \right] + 4 a \tan^2 \left[ \frac{x}{2} \right]} \right)$$

**Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cot^2[x]} \, dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$-\sqrt{a-b} \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \cot[x]}{\sqrt{a+b \cot^2[x]}} \right] - \sqrt{b} \operatorname{ArcTanh} \left[ \frac{\sqrt{b} \cot[x]}{\sqrt{a+b \cot^2[x]}} \right]$$

Result (type 3, 167 leaves):

$$\frac{1}{2} \operatorname{Im} \left( \sqrt{a-b} \operatorname{Log} \left[ -\frac{4 \operatorname{Im} \left( a - \operatorname{Im} b \cot[x] + \sqrt{a-b} \sqrt{a+b \cot^2[x]} \right)}{(a-b)^{3/2} (\operatorname{Im} + \cot[x])} \right] - \sqrt{a-b} \operatorname{Log} \left[ \frac{4 \operatorname{Im} \left( a + \operatorname{Im} b \cot[x] + \sqrt{a-b} \sqrt{a+b \cot^2[x]} \right)}{(a-b)^{3/2} (-\operatorname{Im} + \cot[x])} \right] + 2 \operatorname{Im} \sqrt{b} \operatorname{Log} \left[ b \cot[x] + \sqrt{b} \sqrt{a+b \cot^2[x]} \right] \right)$$

**Problem 24: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cot^2[x]} \tan^2[x] \, dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\sqrt{a-b} \operatorname{ArcTan} \left[ \frac{\sqrt{a-b} \cot[x]}{\sqrt{a+b \cot^2[x]}} \right] + \sqrt{a+b \cot^2[x]} \tan[x]$$

Result (type 3, 129 leaves):

$$\left( \sqrt{-(-a-b+(a-b)\cos[2x])} \operatorname{Csc}[x]^2 \left( -2\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a-b}\cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right] + \sqrt{-2(a+b)+2(a-b)\cos[2x]} \operatorname{Sec}[x] \right) \operatorname{Sin}[x] \right) / \left( 2\sqrt{-a-b+(a-b)\cos[2x]} \right)$$

**Problem 26: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[x]^3 (a+b\operatorname{Cot}[x]^2)^{3/2} dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$-(a-b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\operatorname{Cot}[x]^2}}{\sqrt{a-b}}\right] + (a-b)\sqrt{a+b\operatorname{Cot}[x]^2} + \frac{1}{3}(a+b\operatorname{Cot}[x]^2)^{3/2} - \frac{(a+b\operatorname{Cot}[x]^2)^{5/2}}{5b}$$

Result (type 4, 531 leaves):

$$\begin{aligned} & \sqrt{\frac{-a-b+a\cos[2x]-b\cos[2x]}{-1+\cos[2x]}} \left( -\frac{3a^2-26ab+23b^2}{15b} + \frac{1}{15}(-6a+11b)\operatorname{Csc}[x]^2 - \frac{1}{5}b\operatorname{Csc}[x]^4 \right) + \\ & \left( 2i(a-b)^2(1+\cos[x])\sqrt{\frac{-1+\cos[2x]}{(1+\cos[x])^2}}\sqrt{\frac{-a-b+(a-b)\cos[2x]}{-1+\cos[2x]}} \right. \\ & \left( \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] - \right. \\ & \left. 2\operatorname{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] \right) \tan\left[\frac{x}{2}\right] \\ & \left. \sqrt{1+\frac{b\tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\sqrt{1-\frac{b\tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \right) / \\ & \left( \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\sqrt{-a-b+(a-b)\cos[2x]}\sqrt{-\tan\left[\frac{x}{2}\right]^2}\left(1+\tan\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4a\tan\left[\frac{x}{2}\right]^2+b(-1+\tan\left[\frac{x}{2}\right]^2)^2}{(1+\tan\left[\frac{x}{2}\right]^2)^2}} \right) \right) \end{aligned}$$

**Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cot}[x] (a + b \text{Cot}[x]^2)^{3/2} dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$(a - b)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a + b \text{Cot}[x]^2}}{\sqrt{a - b}}\right] - (a - b) \sqrt{a + b \text{Cot}[x]^2} - \frac{1}{3} (a + b \text{Cot}[x]^2)^{3/2}$$

Result (type 4, 503 leaves):

$$\begin{aligned} & \sqrt{\frac{-a - b + a \text{Cos}[2x] - b \text{Cos}[2x]}{-1 + \text{Cos}[2x]}} \left( -\frac{4}{3} (a - b) - \frac{1}{3} b \text{Csc}[x]^2 \right) - \\ & \left( 2 i (a - b)^2 (1 + \text{Cos}[x]) \sqrt{\frac{-1 + \text{Cos}[2x]}{(1 + \text{Cos}[x])^2}} \sqrt{\frac{-a - b + (a - b) \text{Cos}[2x]}{-1 + \text{Cos}[2x]}} \left( \text{EllipticF}\left[ i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a + 2\sqrt{a(a-b)} - b}} \tan\left[\frac{x}{2}\right]\right], \right. \right. \right. \\ & \left. \left. \frac{-2a - 2\sqrt{a(a-b)} + b}{-2a + 2\sqrt{a(a-b)} + b} \right] - 2 \text{EllipticPi}\left[ \frac{2a + 2\sqrt{a(a-b)} - b}{b}, i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a + 2\sqrt{a(a-b)} - b}} \tan\left[\frac{x}{2}\right]\right], \frac{-2a - 2\sqrt{a(a-b)} + b}{-2a + 2\sqrt{a(a-b)} + b} \right] \right) \\ & \tan\left[\frac{x}{2}\right] \sqrt{1 + \frac{b \tan\left[\frac{x}{2}\right]^2}{2a + 2\sqrt{a(a-b)} - b}} \sqrt{1 - \frac{b \tan\left[\frac{x}{2}\right]^2}{-2a + 2\sqrt{a(a-b)} + b}} \Big/ \\ & \left( \sqrt{\frac{b}{2a + 2\sqrt{a(a-b)} - b}} \sqrt{-a - b + (a - b) \text{Cos}[2x]} \sqrt{-\tan\left[\frac{x}{2}\right]^2} \left(1 + \tan\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4a \tan\left[\frac{x}{2}\right]^2 + b \left(-1 + \tan\left[\frac{x}{2}\right]^2\right)^2}{\left(1 + \tan\left[\frac{x}{2}\right]^2\right)^2}} \right) \end{aligned}$$

**Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \text{Cot}[x]^2)^{3/2} \text{Tan}[x] dx$$

Optimal (type 3, 75 leaves, 8 steps):

$$a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a + b \text{Cot}[x]^2}}{\sqrt{a}}\right] - (a - b)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a + b \text{Cot}[x]^2}}{\sqrt{a - b}}\right] - b \sqrt{a + b \text{Cot}[x]^2}$$

Result (type 3, 230 leaves):

$$-\frac{b\sqrt{(a+b+(-a+b)\cos[2x])\csc[x]^2}}{\sqrt{2}} + \frac{1}{2\sqrt{a-b}\sqrt{b+a\tan[x]^2}}\sqrt{a+b\cot[x]^2} \left( 2a^{3/2}\sqrt{a-b}\log[a\tan[x]+\sqrt{a}\sqrt{b+a\tan[x]^2}] + \right. \\ \left. (a-b)^2 \left( \log\left[\frac{4(b+i a\tan[x]-i\sqrt{a-b}\sqrt{b+a\tan[x]^2})}{(a-b)^{5/2}(-i+\tan[x])}\right] - \log\left[\frac{4i(i b+a\tan[x]+\sqrt{a-b}\sqrt{b+a\tan[x]^2})}{(a-b)^{5/2}(i+\tan[x])}\right] \right) \right) \tan[x]$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int (a+b\cot[x]^2)^{3/2}\tan[x]^2 dx$$

Optimal (type 3, 80 leaves, 7 steps):

$$(a-b)^{3/2}\operatorname{ArcTan}\left[\frac{\sqrt{a-b}\cot[x]}{\sqrt{a+b\cot[x]^2}}\right] - b^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{b}\cot[x]}{\sqrt{a+b\cot[x]^2}}\right] + a\sqrt{a+b\cot[x]^2}\tan[x]$$

Result (type 3, 222 leaves):

$$\left( \sqrt{-(-a-b+(a-b)\cos[2x])\csc[x]^2} \left( -\sqrt{2}(a-b)^2\sqrt{-b}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a-b}\cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right] + \right. \right. \\ \left. \left. \sqrt{a-b} \left( \sqrt{2}b^2\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{-b}\cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right] + a\sqrt{-b}\sqrt{-a-b+(a-b)\cos[2x]}\operatorname{Sec}[x] \right) \right) \right) \\ \left. \sin[x] \right) / \left( \sqrt{2}\sqrt{a-b}\sqrt{-b}\sqrt{-a-b+(a-b)\cos[2x]} \right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int (a+b\cot[c+dx]^2)^{5/2} dx$$

Optimal (type 3, 171 leaves, 8 steps):

$$\frac{(a-b)^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right] - \sqrt{b} (15a^2 - 20ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right]}{d} - \frac{8d}{(7a-4b) b \operatorname{Cot}[c+dx] \sqrt{a+b \operatorname{Cot}[c+dx]^2} - \frac{b \operatorname{Cot}[c+dx] (a+b \operatorname{Cot}[c+dx]^2)^{3/2}}{4d}}$$

Result (type 3, 259 leaves):

$$\frac{1}{8d} \left( b \operatorname{Cot}[c+dx] \sqrt{a+b \operatorname{Cot}[c+dx]^2} (9a-4b+2b \operatorname{Cot}[c+dx]^2) - 4i (a-b)^{5/2} \operatorname{Log}\left[-\frac{4i \left( a - i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2} \right)}{(a-b)^{7/2} (i + \operatorname{Cot}[c+dx])}\right] + \right. \\ \left. 4i (a-b)^{5/2} \operatorname{Log}\left[\frac{4i \left( a + i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2} \right)}{(a-b)^{7/2} (-i + \operatorname{Cot}[c+dx])}\right] + \right. \\ \left. \sqrt{b} (15a^2 - 20ab + 8b^2) \operatorname{Log}\left[ b \operatorname{Cot}[c+dx] + \sqrt{b} \sqrt{a+b \operatorname{Cot}[c+dx]^2} \right] \right)$$

**Problem 32: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+b \operatorname{Cot}[c+dx]^2)^{3/2} dx$$

Optimal (type 3, 126 leaves, 7 steps):

$$\frac{(a-b)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right] - (3a-2b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right] - \frac{b \operatorname{Cot}[c+dx] \sqrt{a+b \operatorname{Cot}[c+dx]^2}}{2d}}{d} - \frac{2d}{2d}$$

Result (type 3, 234 leaves):

$$\frac{1}{2d} \left( -b \operatorname{Cot}[c+dx] \sqrt{a+b \operatorname{Cot}[c+dx]^2} + i (a-b)^{3/2} \operatorname{Log}\left[-\frac{4i \left( a - i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2} \right)}{(a-b)^{5/2} (i + \operatorname{Cot}[c+dx])}\right] - \right. \\ \left. i (a-b)^{3/2} \operatorname{Log}\left[\frac{4i \left( a + i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2} \right)}{(a-b)^{5/2} (-i + \operatorname{Cot}[c+dx])}\right] + \sqrt{b} (-3a+2b) \operatorname{Log}\left[ b \operatorname{Cot}[c+dx] + \sqrt{b} \sqrt{a+b \operatorname{Cot}[c+dx]^2} \right] \right)$$



**Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cot [c + d x]^2} dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$\frac{\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \cot [c+d x]}{\sqrt{a+b \cot [c+d x]^2}}\right]}{d} - \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cot [c+d x]}{\sqrt{a+b \cot [c+d x]^2}}\right]}{d}$$

Result (type 3, 202 leaves):

$$\frac{1}{2 d} i \left( \sqrt{a-b} \operatorname{Log}\left[-\frac{4 i\left(a-i b \cot [c+d x]+\sqrt{a-b} \sqrt{a+b \cot [c+d x]^2}\right)}{(a-b)^{3 / 2}(i+\cot [c+d x])}\right]-\sqrt{a-b} \operatorname{Log}\left[\frac{4 i\left(a+i b \cot [c+d x]+\sqrt{a-b} \sqrt{a+b \cot [c+d x]^2}\right)}{(a-b)^{3 / 2}(-i+\cot [c+d x])}\right]+2 i \sqrt{b} \operatorname{Log}\left[b \cot [c+d x]+\sqrt{b} \sqrt{a+b \cot [c+d x]^2}\right]\right)$$

**Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + b \cot [c + d x]^2}} dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a-b} \cot [c+d x]}{\sqrt{a+b \cot [c+d x]^2}}\right]}{\sqrt{a-b} d}$$

Result (type 3, 151 leaves):

$$\frac{i \left( \operatorname{Log}\left[-\frac{4 i\left(a-i b \cot [c+d x]+\sqrt{a-b} \sqrt{a+b \cot [c+d x]^2}\right)}{\sqrt{a-b}(i+\cot [c+d x])}\right]-\operatorname{Log}\left[\frac{4 i\left(a+i b \cot [c+d x]+\sqrt{a-b} \sqrt{a+b \cot [c+d x]^2}\right)}{\sqrt{a-b}(-i+\cot [c+d x])}\right]\right)}{2 \sqrt{a-b} d}$$

**Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Cot}[c + d x])^{3/2}} dx$$

Optimal (type 3, 85 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right]}{(a-b)^{3/2} d} + \frac{b \operatorname{Cot}[c+dx]}{a(a-b) d \sqrt{a+b \operatorname{Cot}[c+dx]^2}}$$

Result (type 3, 189 leaves):

$$\frac{1}{2d} \left( \frac{2b \operatorname{Cot}[c+dx]}{a(a-b) \sqrt{a+b \operatorname{Cot}[c+dx]^2}} + \frac{i \left( \operatorname{Log}\left[-\frac{4i\sqrt{a-b} \left(a-i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2}\right)}{i+\operatorname{Cot}[c+dx]}\right] - \operatorname{Log}\left[\frac{4i\sqrt{a-b} \left(a+i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2}\right)}{-i+\operatorname{Cot}[c+dx]}\right] \right)}{(a-b)^{3/2}} \right)$$

**Problem 36: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + b \operatorname{Cot}[c + d x])^{5/2}} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right]}{(a-b)^{5/2} d} + \frac{b \operatorname{Cot}[c+dx]}{3a(a-b) d (a+b \operatorname{Cot}[c+dx]^2)^{3/2}} + \frac{(5a-2b) b \operatorname{Cot}[c+dx]}{3a^2(a-b)^2 d \sqrt{a+b \operatorname{Cot}[c+dx]^2}}$$

Result (type 3, 229 leaves):

$$\frac{1}{2d} \left( \frac{2b \operatorname{Cot}[c+dx] \left(3a(2a-b) + (5a-2b) b \operatorname{Cot}[c+dx]^2\right)}{3a^2(a-b)^2 (a+b \operatorname{Cot}[c+dx]^2)^{3/2}} + \frac{i \operatorname{Log}\left[-\frac{4i(a-b)^{3/2} \left(a-i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2}\right)}{i+\operatorname{Cot}[c+dx]}\right]}{(a-b)^{5/2}} - \frac{i \operatorname{Log}\left[\frac{4i(a-b)^{3/2} \left(a+i b \operatorname{Cot}[c+dx] + \sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+dx]^2}\right)}{-i+\operatorname{Cot}[c+dx]}\right]}{(a-b)^{5/2}} \right)$$

**Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Cot}[c + d x])^2} dx$$

Optimal (type 3, 190 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Cot}[c+dx]}{\sqrt{a+b \operatorname{Cot}[c+dx]^2}}\right]}{(a-b)^{7/2} d} + \frac{b \operatorname{Cot}[c+dx]}{5 a (a-b) d (a+b \operatorname{Cot}[c+dx])^{5/2}} + \frac{(9 a-4 b) b \operatorname{Cot}[c+dx]}{15 a^2 (a-b)^2 d (a+b \operatorname{Cot}[c+dx])^{3/2}} + \frac{b (33 a^2-26 a b+8 b^2) \operatorname{Cot}[c+dx]}{15 a^3 (a-b)^3 d \sqrt{a+b \operatorname{Cot}[c+dx]^2}}$$

Result (type 3, 478 leaves):

$$\frac{\sqrt{a+b \operatorname{Cot}[c+dx]^2} \left( -\frac{b \operatorname{Cot}[c+dx]}{5 a (a-b) (a+b \operatorname{Cot}[c+dx])^3} - \frac{(9 a-4 b) b \operatorname{Cot}[c+dx]}{15 a^2 (a-b)^2 (a+b \operatorname{Cot}[c+dx])^2} - \frac{b (33 a^2-26 a b+8 b^2) \operatorname{Cot}[c+dx]}{15 a^3 (a-b)^3 (a+b \operatorname{Cot}[c+dx])^2} \right)}{d} +$$

$$\frac{i \operatorname{Log}\left[ \frac{4 (i a^4-3 i a^3 b+3 i a^2 b^2-i a b^3-a^3 b \operatorname{Cot}[c+dx]+3 a^2 b^2 \operatorname{Cot}[c+dx]-3 a b^3 \operatorname{Cot}[c+dx]+b^4 \operatorname{Cot}[c+dx])}{\sqrt{a-b} (-i+\operatorname{Cot}[c+dx])} + \frac{4 i (a-b)^3 \sqrt{a+b \operatorname{Cot}[c+dx]^2}}{-i+\operatorname{Cot}[c+dx]} \right]}{2 (a-b)^{7/2} d} +$$

$$\frac{i \operatorname{Log}\left[ \frac{4 (-i a^4+3 i a^3 b-3 i a^2 b^2+i a b^3-a^3 b \operatorname{Cot}[c+dx]+3 a^2 b^2 \operatorname{Cot}[c+dx]-3 a b^3 \operatorname{Cot}[c+dx]+b^4 \operatorname{Cot}[c+dx])}{\sqrt{a-b} (i+\operatorname{Cot}[c+dx])} - \frac{4 i (a-b)^3 \sqrt{a+b \operatorname{Cot}[c+dx]^2}}{i+\operatorname{Cot}[c+dx]} \right]}{2 (a-b)^{7/2} d}$$

**Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (1 - \operatorname{Cot}[x]^2)^{3/2} dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$\frac{5}{2} \operatorname{ArcSin}[\operatorname{Cot}[x]] - 2 \sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2} \operatorname{Cot}[x]}{\sqrt{1 - \operatorname{Cot}[x]^2}}\right] + \frac{1}{2} \operatorname{Cot}[x] \sqrt{1 - \operatorname{Cot}[x]^2}$$

Result (type 3, 123 leaves):

$$\frac{1}{2} (1 - \operatorname{Cot}[x]^2)^{3/2} \operatorname{Sec}[2 x]^2 \left( \operatorname{ArcTan}\left[\frac{\operatorname{Cos}[x]}{\sqrt{-\operatorname{Cos}[2 x]}}\right] \sqrt{-\operatorname{Cos}[2 x]} \operatorname{Sin}[x]^3 + \right.$$

$$\left. 4 \operatorname{ArcTanh}\left[\frac{\operatorname{Cos}[x]}{\sqrt{\operatorname{Cos}[2 x]}}\right] \sqrt{\operatorname{Cos}[2 x]} \operatorname{Sin}[x]^3 - 4 \sqrt{2} \sqrt{\operatorname{Cos}[2 x]} \operatorname{Log}\left[\sqrt{2} \operatorname{Cos}[x] + \sqrt{\operatorname{Cos}[2 x]}\right] \operatorname{Sin}[x]^3 - \frac{1}{4} \operatorname{Sin}[4 x] \right)$$

**Problem 44: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[x]^3}{\sqrt{a + b \text{Cot}[x]^2}} dx$$

Optimal (type 3, 52 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{\sqrt{a-b}} - \frac{\sqrt{a+b \text{Cot}[x]^2}}{b}$$

Result (type 4, 481 leaves):

$$-\frac{\sqrt{\frac{-a-b+a \cos[2x]-b \cos[2x]}{-1+\cos[2x]}}}{b} +$$

$$\left( 2 i (1 + \cos[x]) \sqrt{\frac{-1 + \cos[2x]}{(1 + \cos[x])^2}} \sqrt{\frac{-a - b + (a - b) \cos[2x]}{-1 + \cos[2x]}} \left( \text{EllipticF}\left[ i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a + 2\sqrt{a(a-b)} - b}} \tan\left[\frac{x}{2}\right]\right], \right. \right. \right.$$

$$\left. \left. \frac{-2a - 2\sqrt{a(a-b)} + b}{-2a + 2\sqrt{a(a-b)} + b} \right] - 2 \text{EllipticPi}\left[ \frac{2a + 2\sqrt{a(a-b)} - b}{b}, i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a + 2\sqrt{a(a-b)} - b}} \tan\left[\frac{x}{2}\right]\right], \frac{-2a - 2\sqrt{a(a-b)} + b}{-2a + 2\sqrt{a(a-b)} + b} \right] \right)$$

$$\tan\left[\frac{x}{2}\right] \sqrt{1 + \frac{b \tan\left[\frac{x}{2}\right]^2}{2a + 2\sqrt{a(a-b)} - b}} \sqrt{1 - \frac{b \tan\left[\frac{x}{2}\right]^2}{-2a + 2\sqrt{a(a-b)} + b}} \Big/$$

$$\left( \sqrt{\frac{b}{2a + 2\sqrt{a(a-b)} - b}} \sqrt{-a - b + (a - b) \cos[2x]} \sqrt{-\tan\left[\frac{x}{2}\right]^2} \left(1 + \tan\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4a \tan\left[\frac{x}{2}\right]^2 + b \left(-1 + \tan\left[\frac{x}{2}\right]^2\right)^2}{\left(1 + \tan\left[\frac{x}{2}\right]^2\right)^2}} \right)$$

**Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[x]^2}{\sqrt{a + b \text{Cot}[x]^2}} dx$$

Optimal (type 3, 64 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a-b} \cot[x]}{\sqrt{a+b \cot[x]^2}}\right]}{\sqrt{a-b}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{b} \cot[x]}{\sqrt{a+b \cot[x]^2}}\right]}{\sqrt{b}}$$

Result (type 3, 158 leaves):

$$\left( \left( -\sqrt{-b} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a-b} \cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right] + \sqrt{a-b} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{-b} \cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right] \right) \sqrt{(a+b+(-a+b)\cos[2x]) \csc[x]^2 \sin[x]} \right) / \left( \sqrt{a-b} \sqrt{-b} \sqrt{-a-b+(a-b)\cos[2x]} \right)$$

**Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[x]}{\sqrt{a+b \cot[x]^2}} dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a-b}}\right]}{\sqrt{a-b}}$$

Result (type 4, 352 leaves):

$$\left( 2 i \cos\left[\frac{x}{2}\right] (1 + \cos[x]) \sqrt{-(-a-b+(a-b)\cos[2x]) \csc[x]^2} \right. \\ \left( \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] - \right. \\ \left. 2 \text{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b}, i \text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] \right) \sin\left[\frac{x}{2}\right] \\ \left. \sqrt{1 + \frac{b \tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}} \sqrt{1 - \frac{b \tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \right) / \left( \sqrt{\frac{b}{4a+4\sqrt{a(a-b)}-2b}} (a+b+(-a+b)\cos[2x]) \right)$$

### Problem 47: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]}{\sqrt{a + b \cot[x]^2}} dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a-b}}\right]}{\sqrt{a-b}}$$

Result (type 3, 204 leaves):

$$\left( 2 \sqrt{\cos[x]^2} \sqrt{-(-a-b+(a-b)\cos[2x])} \operatorname{Csc}[x]^2 \right. \\ \left. \left( \sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{-\sin[x]^2}}{\sqrt{-b \cos[x]^2 - a \sin[x]^2}}\right] - \sqrt{a} \operatorname{Log}\left[a \sqrt{-1+\cos[2x]} - b \sqrt{-1+\cos[2x]} + \sqrt{a-b} \sqrt{-a-b+(a-b)\cos[2x]}\right] \right) \right) \\ \left. \sqrt{-\sin[x]^4} \right) / \left( \sqrt{a} \sqrt{a-b} \sqrt{-a-b+(a-b)\cos[2x]} \sqrt{\sin[2x]^2} \right)$$

### Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]^2}{\sqrt{a + b \cot[x]^2}} dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a-b} \cot[x]}{\sqrt{a+b \cot[x]^2}}\right]}{\sqrt{a-b}} + \frac{\sqrt{a+b \cot[x]^2} \tan[x]}{a}$$

Result (type 3, 149 leaves):

$$\left( \sqrt{-(-a-b+(a-b)\cos[2x])} \operatorname{Csc}[x]^2 \left( -\sqrt{2} a \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a-b} \cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right] \sin[x] + \sqrt{a-b} \sqrt{-a-b+(a-b)\cos[2x]} \tan[x] \right) \right) / \left( \sqrt{2} a \sqrt{a-b} \sqrt{-a-b+(a-b)\cos[2x]} \right)$$

**Problem 49: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[x]^3}{(a + b \text{Cot}[x]^2)^{3/2}} dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}} + \frac{a}{(a-b)b\sqrt{a+b \text{Cot}[x]^2}}$$

Result (type 4, 489 leaves):

$$-\frac{1}{(a-b)b\sqrt{\frac{b}{4a+4\sqrt{a(a-b)}-2b}}(a+b+(-a+b)\cos[2x])} 4i\cos\left[\frac{x}{2}\right]^2\sqrt{-(-a-b+(a-b)\cos[2x])}\csc[x]^2\sin\left[\frac{x}{2}\right]$$

$$\left(i a \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\sin\left[\frac{x}{2}\right] + b\cos\left[\frac{x}{2}\right] \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]\right)$$

$$\sqrt{1+\frac{b\tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\sqrt{1-\frac{b\tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}}-2b\cos\left[\frac{x}{2}\right] \text{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b},\right.$$

$$\left.i \text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]\sqrt{1+\frac{b\tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\sqrt{1-\frac{b\tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}}$$

**Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[x]^2}{(a + b \text{Cot}[x]^2)^{3/2}} dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a-b}\text{Cot}[x]}{\sqrt{a+b \text{Cot}[x]^2}}\right]}{(a-b)^{3/2}} - \frac{\text{Cot}[x]}{(a-b)\sqrt{a+b \text{Cot}[x]^2}}$$

Result (type 3, 157 leaves):

$$\left( -2\sqrt{a-b}\sqrt{-a-b+(a-b)\cos[2x]}\cot[x] + \sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a-b}\cos[x]}{\sqrt{-a-b+(a-b)\cos[2x]}}\right](-a-b+(a-b)\cos[2x])\csc[x] \right) /$$

$$\left( (a-b)^{3/2}\sqrt{-2(a+b)+2(a-b)\cos[2x]}\sqrt{-(-a-b+(a-b)\cos[2x])\csc[x]^2} \right)$$

**Problem 51: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[x]}{(a+b\cot[x]^2)^{3/2}} dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b\cot[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}} - \frac{1}{(a-b)\sqrt{a+b\cot[x]^2}}$$

Result (type 4, 483 leaves):

$$-\frac{1}{(a-b)\sqrt{\frac{b}{4a+4\sqrt{a(a-b)}-2b}}(a+b+(-a+b)\cos[2x])} - 4\cos\left[\frac{x}{2}\right]^2\sqrt{-(-a-b+(a-b)\cos[2x])\csc[x]^2}\sin\left[\frac{x}{2}\right]$$

$$\left( \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\sin\left[\frac{x}{2}\right] - i\cos\left[\frac{x}{2}\right] \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] \right.$$

$$\left. \sqrt{1+\frac{b\tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\sqrt{1-\frac{b\tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} + 2i\cos\left[\frac{x}{2}\right] \operatorname{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b}, \right.$$

$$\left. i\operatorname{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] \sqrt{1+\frac{b\tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\sqrt{1-\frac{b\tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \right)$$

**Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[x]}{(a+b\cot[x]^2)^{3/2}} dx$$

Optimal (type 3, 84 leaves, 8 steps):



$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}} + \frac{b}{a(a-b)\sqrt{a+b \cot[x]^2}}$$

Result (type 3, 243 leaves):

$$\frac{\sqrt{2} b}{a(a-b)\sqrt{(a+b+(-a+b)\cos[2x])\csc[x]^2}} + \left( \cot[x] \left( 2(a-b)^{3/2} \log[a \tan[x] + \sqrt{a}\sqrt{b+a \tan[x]^2}] + a^{3/2} \left( \log\left[\frac{4i(i b - a \tan[x] + \sqrt{a-b}\sqrt{b+a \tan[x]^2})}{a\sqrt{a-b}(-i + \tan[x])}\right] - \log\left[\frac{4(b-i(a \tan[x] + \sqrt{a-b}\sqrt{b+a \tan[x]^2}))}{a\sqrt{a-b}(i + \tan[x])}\right] \right) \right) \sqrt{b+a \tan[x]^2} \right) / \left( 2 a^{3/2} (a-b)^{3/2} \sqrt{a+b \cot[x]^2} \right)$$

**Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[x]^3}{(a+b \cot[x]^2)^{5/2}} dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \cot[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{5/2}} + \frac{a}{3(a-b)b(a+b \cot[x]^2)^{3/2}} + \frac{1}{(a-b)^2 \sqrt{a+b \cot[x]^2}}$$

Result (type 4, 579 leaves):

$$\begin{aligned}
& \sqrt{\frac{-a-b+a\cos[2x]-b\cos[2x]}{-1+\cos[2x]}} \left( \frac{a+3b}{3(a-b)^3 b} + \frac{4ab}{3(a-b)^3 (-a-b+a\cos[2x]-b\cos[2x])^2} + \frac{2(2a+3b)}{3(a-b)^3 (-a-b+a\cos[2x]-b\cos[2x])} \right) + \\
& \left( 2i(1+\cos[x]) \sqrt{\frac{-1+\cos[2x]}{(1+\cos[x])^2}} \sqrt{\frac{-a-b+(a-b)\cos[2x]}{-1+\cos[2x]}} \right. \\
& \left( \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] - \right. \\
& \left. 2 \text{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b}, i \text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right]\right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right] \right) \tan\left[\frac{x}{2}\right] \\
& \sqrt{1+\frac{b \tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}} \sqrt{1-\frac{b \tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \Big/ \\
& \left( (a-b)^2 \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \sqrt{-a-b+(a-b)\cos[2x]} \sqrt{-\tan\left[\frac{x}{2}\right]^2} \left(1+\tan\left[\frac{x}{2}\right]^2\right) \sqrt{-\frac{4a \tan\left[\frac{x}{2}\right]^2+b(-1+\tan\left[\frac{x}{2}\right]^2)^2}{(1+\tan\left[\frac{x}{2}\right]^2)^2}} \right)
\end{aligned}$$

**Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[x]^2}{(a+b\cot[x]^2)^{5/2}} dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a-b}\cot[x]}{\sqrt{a+b\cot[x]^2}}\right]}{(a-b)^{5/2}} - \frac{\cot[x]}{3(a-b)(a+b\cot[x]^2)^{3/2}} - \frac{(2a+b)\cot[x]}{3a(a-b)^2\sqrt{a+b\cot[x]^2}}$$

Result (type 3, 194 leaves):

$$- \left( \left( \left( 6 \sqrt{2} a \operatorname{ArcTanh} \left[ \frac{\sqrt{2} \sqrt{a-b} \cos [x]}{\sqrt{-a-b+(a-b) \cos [2 x]}} \right] (a+b+(-a+b) \cos [2 x])^2 + \right. \right. \right. \\ \left. \left. \left. 2 \sqrt{a-b} \sqrt{-a-b+(a-b) \cos [2 x]} \left( 3 (a+b)^2 \cos [x] + (-3 a^2+2 a b+b^2) \cos [3 x] \right) \right) \right) \right. \\ \left. \left. \sqrt{-(-a-b+(a-b) \cos [2 x]) \operatorname{Csc}[x]^2 \sin [x]} \right) / \left( 6 \sqrt{2} a (a-b)^{5/2} (-a-b+(a-b) \cos [2 x])^{5/2} \right)$$

**Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cot [x]}{(a+b \cot [x]^2)^{5/2}} dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cot [x]^2}}{\sqrt{a-b}} \right]}{(a-b)^{5/2}} - \frac{1}{3(a-b)(a+b \cot [x]^2)^{3/2}} - \frac{1}{(a-b)^2 \sqrt{a+b \cot [x]^2}}$$

Result (type 4, 566 leaves):

$$\sqrt{\frac{-a-b+a\cos[2x]-b\cos[2x]}{-1+\cos[2x]}} \left( -\frac{4}{3(a-b)^3} - \frac{4b^2}{3(a-b)^3(-a-b+a\cos[2x]-b\cos[2x])^2} - \frac{10b}{3(a-b)^3(-a-b+a\cos[2x]-b\cos[2x])} \right) -$$

$$\left( 2i(1+\cos[x]) \sqrt{\frac{-1+\cos[2x]}{(1+\cos[x])^2}} \sqrt{\frac{-a-b+(a-b)\cos[2x]}{-1+\cos[2x]}} \right.$$

$$\left( \text{EllipticF}\left[ i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right] \right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b} \right] - \right.$$

$$\left. 2 \text{EllipticPi}\left[ \frac{2a+2\sqrt{a(a-b)}-b}{b}, i \text{ArcSinh}\left[ \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \tan\left[\frac{x}{2}\right] \right], \frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b} \right] \right) \tan\left[\frac{x}{2}\right]$$

$$\sqrt{1+\frac{b\tan\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}} \sqrt{1-\frac{b\tan\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}} \Big/$$

$$\left( (a-b)^2 \sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}} \sqrt{-a-b+(a-b)\cos[2x]} \sqrt{-\tan\left[\frac{x}{2}\right]^2} \left(1+\tan\left[\frac{x}{2}\right]^2\right) \sqrt{-\frac{4a\tan\left[\frac{x}{2}\right]^2+b(-1+\tan\left[\frac{x}{2}\right]^2)^2}{(1+\tan\left[\frac{x}{2}\right]^2)^2}} \right)$$

**Problem 57:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]}{(a+b\cot[x]^2)^{5/2}} dx$$

Optimal (type 3, 118 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\cot[x]^2}}{\sqrt{a}}\right]}{a^{5/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\cot[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{5/2}} + \frac{b}{3a(a-b)(a+b\cot[x]^2)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot[x]^2}}$$

Result (type 3, 982 leaves):

$$\begin{aligned}
& \sqrt{\frac{-a-b+a\cos[2x]-b\cos[2x]}{-1+\cos[2x]}} \left( \frac{(7a-3b)b}{3a^2(a-b)^3} + \frac{4b^3}{3a(a-b)^3(-a-b+a\cos[2x]-b\cos[2x])^2} + \frac{2(8a-3b)b^2}{3a^2(a-b)^3(-a-b+a\cos[2x]-b\cos[2x])} \right) + \\
& \left( \sqrt{\frac{-a-b+a\cos[2x]-b\cos[2x]}{-1+\cos[2x]}} (-i+\cot[x])(i+\cot[x])(a+b\cot[x]^2) \left( 2(a-b)^{5/2} \operatorname{Log}[a\tan[x]+\sqrt{a-b}\sqrt{b+a\tan[x]^2}] + \right. \right. \\
& \left. \left. a^{5/2} \left( \operatorname{Log}\left[ \frac{4(b+ia\tan[x]-i\sqrt{a-b}\sqrt{b+a\tan[x]^2})}{a^2\sqrt{a-b}(-i+\tan[x])} \right] - \operatorname{Log}\left[ \frac{4i(b+a\tan[x]+\sqrt{a-b}\sqrt{b+a\tan[x]^2})}{a^2\sqrt{a-b}(i+\tan[x])} \right] \right) \right) \\
& \left. (-3a^2+8ab-4b^2+a^2\csc[x]\sin[3x])\tan[x] \left( -a+ib\cot[x]+\sqrt{a-b}\cot[x]\sqrt{b+a\tan[x]^2} \right) \right. \\
& \left. \left( a+ib\cot[x]+\sqrt{a-b}\cot[x]\sqrt{b+a\tan[x]^2} \right) \right) / \\
& \left( 4a^{5/2}(a-b)^2(-a-b+a\cos[2x]-b\cos[2x]) \left( 2ia^4b\csc[x]^2-6ia^3b^2\csc[x]^2+6ia^2b^3\csc[x]^2-2iab^4\csc[x]^2- \right. \right. \\
& 2ia^3b^2\cot[x]^2\csc[x]^2+4ia^4b^4\cot[x]^2\csc[x]^2-2ib^5\cot[x]^2\csc[x]^2-4ia^2b^3\cot[x]^4\csc[x]^2+ \\
& 6ia^4b^4\cot[x]^4\csc[x]^2-2ib^5\cot[x]^4\csc[x]^2-a^3\sqrt{a-b}b\csc[x]^2\sqrt{b+a\tan[x]^2}+2a^2\sqrt{a-b}b^2\csc[x]^2\sqrt{b+a\tan[x]^2}- \\
& a\sqrt{a-b}b^3\csc[x]^2\sqrt{b+a\tan[x]^2}+a^3\sqrt{a-b}b\cot[x]^2\csc[x]^2\sqrt{b+a\tan[x]^2}-2a^2\sqrt{a-b}b^2\cot[x]^2\csc[x]^2\sqrt{b+a\tan[x]^2}+ \\
& 4a\sqrt{a-b}b^3\cot[x]^2\csc[x]^2\sqrt{b+a\tan[x]^2}-2\sqrt{a-b}b^4\cot[x]^2\csc[x]^2\sqrt{b+a\tan[x]^2}-2a^2\sqrt{a-b}b^2\cot[x]^4 \\
& \left. \left. \csc[x]^2\sqrt{b+a\tan[x]^2}+5a\sqrt{a-b}b^3\cot[x]^4\csc[x]^2\sqrt{b+a\tan[x]^2}-2\sqrt{a-b}b^4\cot[x]^4\csc[x]^2\sqrt{b+a\tan[x]^2} \right) \right) \right)
\end{aligned}$$

**Problem 60: Result more than twice size of optimal antiderivative.**

$$\int \cot[x] \sqrt{a+b\cot[x]^4} dx$$

Optimal (type 3, 90 leaves, 8 steps):

$$\frac{1}{2} \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b}\cot[x]^2}{\sqrt{a+b\cot[x]^4}}\right] + \frac{1}{2} \sqrt{a+b} \operatorname{ArcTanh}\left[\frac{a-b\cot[x]^2}{\sqrt{a+b}\sqrt{a+b\cot[x]^4}}\right] - \frac{1}{2} \sqrt{a+b\cot[x]^4}$$

Result (type 3, 1081 leaves):

$$\begin{aligned}
& -\frac{1}{2} \sqrt{\frac{3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]}} + \\
& \left( \sqrt{\frac{-3a - 3b + 4a \cos[2x] - 4b \cos[2x] - a \cos[4x] - b \cos[4x]}{-3 + 4 \cos[2x] - \cos[4x]}} \operatorname{Cot}[x]^3 (a + b \operatorname{Cot}[x]^4) \right. \\
& \left. \left( -\sqrt{a+b} \operatorname{Log}[\operatorname{Sec}[x]^2] + \sqrt{b} \operatorname{Log}[\operatorname{Tan}[x]^2] - \sqrt{b} \operatorname{Log}[b + \sqrt{b} \sqrt{b+a \operatorname{Tan}[x]^4}] + \sqrt{a+b} \operatorname{Log}[b - a \operatorname{Tan}[x]^2 + \sqrt{a+b} \sqrt{b+a \operatorname{Tan}[x]^4}] \right) \right. \\
& \left. \left( 2a \operatorname{Sin}[2x] - 2b \operatorname{Sin}[2x] - a \operatorname{Sin}[4x] - b \operatorname{Sin}[4x] \right) \left( \sqrt{b} + \sqrt{b+a \operatorname{Tan}[x]^4} \right) \left( a - b \operatorname{Cot}[x]^2 - \sqrt{a+b} \operatorname{Cot}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} \right) \right) / \\
& \left( 2(-3a - 3b + 4a \cos[2x] - 4b \cos[2x] - a \cos[4x] - b \cos[4x]) \right. \\
& \left. \left( -a^3 - a^2 b + a^2 \sqrt{b} \sqrt{a+b} \operatorname{Cot}[x]^2 - 2a^2 b \operatorname{Cot}[x]^4 - 2a b^2 \operatorname{Cot}[x]^4 - a b^{3/2} \sqrt{a+b} \operatorname{Cot}[x]^4 + a b^{3/2} \sqrt{a+b} \operatorname{Cot}[x]^6 - a b^2 \operatorname{Cot}[x]^8 - b^3 \operatorname{Cot}[x]^8 - \right. \right. \\
& \left. b^{5/2} \sqrt{a+b} \operatorname{Cot}[x]^8 + a^3 \operatorname{Csc}[x]^2 + a^2 b \operatorname{Csc}[x]^2 - a^2 b \operatorname{Cot}[x]^2 \operatorname{Csc}[x]^2 - a^2 \sqrt{b} \sqrt{a+b} \operatorname{Cot}[x]^2 \operatorname{Csc}[x]^2 + a^2 b \operatorname{Cot}[x]^4 \operatorname{Csc}[x]^2 + \right. \\
& \left. 2a b^2 \operatorname{Cot}[x]^4 \operatorname{Csc}[x]^2 + a b^{3/2} \sqrt{a+b} \operatorname{Cot}[x]^4 \operatorname{Csc}[x]^2 - a b^2 \operatorname{Cot}[x]^6 \operatorname{Csc}[x]^2 - a b^{3/2} \sqrt{a+b} \operatorname{Cot}[x]^6 \operatorname{Csc}[x]^2 + b^3 \operatorname{Cot}[x]^8 \operatorname{Csc}[x]^2 + \right. \\
& \left. b^{5/2} \sqrt{a+b} \operatorname{Cot}[x]^8 \operatorname{Csc}[x]^2 + a^2 \sqrt{a+b} \operatorname{Cot}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} - a^2 \sqrt{b} \operatorname{Cot}[x]^4 \sqrt{b+a \operatorname{Tan}[x]^4} - a b^{3/2} \operatorname{Cot}[x]^4 \sqrt{b+a \operatorname{Tan}[x]^4} - \right. \\
& \left. a b \sqrt{a+b} \operatorname{Cot}[x]^4 \sqrt{b+a \operatorname{Tan}[x]^4} + a b \sqrt{a+b} \operatorname{Cot}[x]^6 \sqrt{b+a \operatorname{Tan}[x]^4} - a b^{3/2} \operatorname{Cot}[x]^8 \sqrt{b+a \operatorname{Tan}[x]^4} - b^{5/2} \operatorname{Cot}[x]^8 \sqrt{b+a \operatorname{Tan}[x]^4} - \right. \\
& \left. b^2 \sqrt{a+b} \operatorname{Cot}[x]^8 \sqrt{b+a \operatorname{Tan}[x]^4} - a^2 \sqrt{a+b} \operatorname{Cot}[x]^2 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} + a^2 \sqrt{b} \operatorname{Cot}[x]^4 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} + \right. \\
& \left. a b^{3/2} \operatorname{Cot}[x]^4 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} + a b \sqrt{a+b} \operatorname{Cot}[x]^4 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} - a b^{3/2} \operatorname{Cot}[x]^6 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} - \right. \\
& \left. a b \sqrt{a+b} \operatorname{Cot}[x]^6 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} + b^{5/2} \operatorname{Cot}[x]^8 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} + b^2 \sqrt{a+b} \operatorname{Cot}[x]^8 \operatorname{Csc}[x]^2 \sqrt{b+a \operatorname{Tan}[x]^4} \right) \left. \right)
\end{aligned}$$

### Problem 61: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cot}[x] (a + b \operatorname{Cot}[x]^4)^{3/2} dx$$

Optimal (type 3, 126 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{4} \sqrt{b} (3a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cot}[x]^2}{\sqrt{a + b \operatorname{Cot}[x]^4}}\right] + \\
& \frac{1}{2} (a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{a - b \operatorname{Cot}[x]^2}{\sqrt{a+b} \sqrt{a + b \operatorname{Cot}[x]^4}}\right] - \frac{1}{4} (2(a+b) - b \operatorname{Cot}[x]^2) \sqrt{a + b \operatorname{Cot}[x]^4} - \frac{1}{6} (a + b \operatorname{Cot}[x]^4)^{3/2}
\end{aligned}$$

Result (type 3, 1837 leaves):

$$\begin{aligned}
& \sqrt{\frac{3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]}} \left( \frac{1}{12} (-8a - 11b) + \frac{7}{12} b \operatorname{Csc}[x]^2 - \frac{1}{6} b \operatorname{Csc}[x]^4 \right) + \\
& \left( \sqrt{a + b \cot[x]^4} \left( 2(a + b)^{3/2} \operatorname{Log}[\sec[x]^2] - \sqrt{b} (3a + 2b) \operatorname{Log}[\tan[x]^2] + \right. \right. \\
& \quad \left. \left. \sqrt{b} (3a + 2b) \operatorname{Log}[b + \sqrt{b} \sqrt{b + a \tan[x]^4}] - 2(a + b)^{3/2} \operatorname{Log}[b - a \tan[x]^2 + \sqrt{a + b} \sqrt{b + a \tan[x]^4}] \right) \right) \\
& \left( \left( 2a^2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2x] + \cos[4x]} + \frac{3b}{3 - 4 \cos[2x] + \cos[4x]} - \frac{4a \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{4b \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \right. \right. \right. \\
& \quad \left. \left. \frac{a \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} \right) \operatorname{Sin}[2x] \right) / (3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]) - \\
& \left( 2ab \sqrt{\left( \frac{3a}{3 - 4 \cos[2x] + \cos[4x]} + \frac{3b}{3 - 4 \cos[2x] + \cos[4x]} - \frac{4a \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{4b \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \right. \right. \\
& \quad \left. \left. \frac{a \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} \right) \operatorname{Sin}[2x] \right) / (3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]) - \\
& \left( 2b^2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2x] + \cos[4x]} + \frac{3b}{3 - 4 \cos[2x] + \cos[4x]} - \frac{4a \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{4b \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \right. \right. \\
& \quad \left. \left. \frac{a \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} \right) \operatorname{Sin}[2x] \right) / (3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]) - \\
& \left( a^2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2x] + \cos[4x]} + \frac{3b}{3 - 4 \cos[2x] + \cos[4x]} - \frac{4a \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{4b \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \right. \right. \\
& \quad \left. \left. \frac{a \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} \right) \operatorname{Sin}[4x] \right) / (3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]) - \\
& \left( 2ab \sqrt{\left( \frac{3a}{3 - 4 \cos[2x] + \cos[4x]} + \frac{3b}{3 - 4 \cos[2x] + \cos[4x]} - \frac{4a \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{4b \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \right. \right. \\
& \quad \left. \left. \frac{a \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} \right) \operatorname{Sin}[4x] \right) / (3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]) - \\
& \left( b^2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2x] + \cos[4x]} + \frac{3b}{3 - 4 \cos[2x] + \cos[4x]} - \frac{4a \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{4b \cos[2x]}{3 - 4 \cos[2x] + \cos[4x]} + \right. \right. \\
& \quad \left. \left. \frac{a \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} + \frac{b \cos[4x]}{3 - 4 \cos[2x] + \cos[4x]} \right) \operatorname{Sin}[4x] \right) / (3a + 3b - 4a \cos[2x] + 4b \cos[2x] + a \cos[4x] + b \cos[4x]) \Big) \\
& \operatorname{Tan}[x]^2 \Big) / \left( 4 \sqrt{b + a \tan[x]^4} \left( -\frac{1}{2(b + a \tan[x]^4)^{3/2}} a \sqrt{a + b \cot[x]^4} \left( 2(a + b)^{3/2} \operatorname{Log}[\sec[x]^2] - \sqrt{b} (3a + 2b) \operatorname{Log}[\tan[x]^2] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{b} (3a + 2b) \operatorname{Log}[b + \sqrt{b} \sqrt{b + a \operatorname{Tan}[x]^4}] - 2(a + b)^{3/2} \operatorname{Log}[b - a \operatorname{Tan}[x]^2 + \sqrt{a + b} \sqrt{b + a \operatorname{Tan}[x]^4}] \Big) \operatorname{Sec}[x]^2 \operatorname{Tan}[x]^5 - \\
& \left( b \operatorname{Cot}[x] \operatorname{Csc}[x]^2 \left( 2(a + b)^{3/2} \operatorname{Log}[\operatorname{Sec}[x]^2] - \sqrt{b} (3a + 2b) \operatorname{Log}[\operatorname{Tan}[x]^2] + \sqrt{b} (3a + 2b) \operatorname{Log}[b + \sqrt{b} \sqrt{b + a \operatorname{Tan}[x]^4}] - \right. \right. \\
& \quad \left. \left. 2(a + b)^{3/2} \operatorname{Log}[b - a \operatorname{Tan}[x]^2 + \sqrt{a + b} \sqrt{b + a \operatorname{Tan}[x]^4}] \right) \right) / \left( 2\sqrt{a + b} \operatorname{Cot}[x]^4 \sqrt{b + a \operatorname{Tan}[x]^4} \right) + \frac{1}{2\sqrt{b + a \operatorname{Tan}[x]^4}} \\
& \sqrt{a + b} \operatorname{Cot}[x]^4 \left( 2(a + b)^{3/2} \operatorname{Log}[\operatorname{Sec}[x]^2] - \sqrt{b} (3a + 2b) \operatorname{Log}[\operatorname{Tan}[x]^2] + \sqrt{b} (3a + 2b) \operatorname{Log}[b + \sqrt{b} \sqrt{b + a \operatorname{Tan}[x]^4}] - \right. \\
& \quad \left. 2(a + b)^{3/2} \operatorname{Log}[b - a \operatorname{Tan}[x]^2 + \sqrt{a + b} \sqrt{b + a \operatorname{Tan}[x]^4}] \right) \operatorname{Sec}[x]^2 \operatorname{Tan}[x] + \\
& \frac{1}{4\sqrt{b + a \operatorname{Tan}[x]^4}} \sqrt{a + b} \operatorname{Cot}[x]^4 \operatorname{Tan}[x]^2 \left( -2\sqrt{b} (3a + 2b) \operatorname{Csc}[x] \operatorname{Sec}[x] + 4(a + b)^{3/2} \operatorname{Tan}[x] + \right. \\
& \quad \left. \frac{2ab(3a + 2b) \operatorname{Sec}[x]^2 \operatorname{Tan}[x]^3}{\sqrt{b + a \operatorname{Tan}[x]^4} (b + \sqrt{b} \sqrt{b + a \operatorname{Tan}[x]^4})} - \frac{2(a + b)^{3/2} \left( -2a \operatorname{Sec}[x]^2 \operatorname{Tan}[x] + \frac{2a\sqrt{a + b} \operatorname{Sec}[x]^2 \operatorname{Tan}[x]^3}{\sqrt{b + a \operatorname{Tan}[x]^4}} \right)}{b - a \operatorname{Tan}[x]^2 + \sqrt{a + b} \sqrt{b + a \operatorname{Tan}[x]^4}} \right) \Big) \Big) \Big)
\end{aligned}$$

**Problem 62:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cot}[x]}{\sqrt{a + b \operatorname{Cot}[x]^4}} dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{a - b \operatorname{Cot}[x]^2}{\sqrt{a + b} \sqrt{a + b \operatorname{Cot}[x]^4}}\right]}{2\sqrt{a + b}}$$

Result (type 4, 72807 leaves): Display of huge result suppressed!

**Problem 63:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cot}[x]}{(a + b \operatorname{Cot}[x]^4)^{3/2}} dx$$

Optimal (type 3, 74 leaves, 6 steps):



$$\frac{\text{ArcTanh}\left[\frac{a-b\text{Cot}[x]^2}{\sqrt{a+b}\sqrt{a+b\text{Cot}[x]^4}}\right]}{2(a+b)^{3/2}} - \frac{a+b\text{Cot}[x]^2}{2a(a+b)\sqrt{a+b\text{Cot}[x]^4}}$$

Result (type 4, 61 450 leaves): Display of huge result suppressed!

**Problem 64:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{(a+b\text{Cot}[x]^4)^{5/2}} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a-b\text{Cot}[x]^2}{\sqrt{a+b}\sqrt{a+b\text{Cot}[x]^4}}\right]}{2(a+b)^{5/2}} - \frac{a+b\text{Cot}[x]^2}{6a(a+b)(a+b\text{Cot}[x]^4)^{3/2}} - \frac{3a^2+b(5a+2b)\text{Cot}[x]^2}{6a^2(a+b)^2\sqrt{a+b\text{Cot}[x]^4}}$$

Result (type 4, 73 108 leaves): Display of huge result suppressed!

## Test results for the 32 problems in "4.4.9 trig^m (a+b cot^n+c cot^(2 n))^p.m"

**Problem 1:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d+ex]^5}{\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}} dx$$

Optimal (type 3, 547 leaves, 15 steps):

$$\begin{aligned}
& \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh}\left[\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\cot[d+ex]}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}\right]}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} + \\
& \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh}\left[\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\cot[d+ex]}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}\right]}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} - \frac{b \operatorname{ArcTanh}\left[\frac{b+2c\cot[d+ex]}{2\sqrt{c}\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}\right]}{2c^{3/2}e} + \\
& \frac{b(5b^2-12ac) \operatorname{ArcTanh}\left[\frac{b+2c\cot[d+ex]}{2\sqrt{c}\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}\right]}{16c^{7/2}e} + \frac{\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}{ce} - \\
& \frac{\cot[d+ex]^2\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}{3ce} - \frac{(15b^2-16ac-10bc\cot[d+ex])\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}{24c^3e}
\end{aligned}$$

Result (type 3, 3681 leaves):

$$\begin{aligned}
& \frac{\left(\frac{-15b^2+16ac+32c^2}{24c^3} + \frac{5b\cot[d+ex]}{12c^2} - \frac{\operatorname{Csc}[d+ex]^2}{3c}\right) \sqrt{\frac{-a-ca\cos[2(d+ex)]-c\cos[2(d+ex)]-b\sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{e} + \\
& \left( \left( b\sqrt{a-ib-c}\sqrt{a+ib-c}(-5b^2+4c(3a+2c)) \operatorname{Log}[\operatorname{Tan}[d+ex]] - \right. \right. \\
& \left. \left. 8\sqrt{a+ib-c}c^{7/2} \operatorname{Log}\left[\left(-2c-2ia\operatorname{Tan}[d+ex]-b(i+\operatorname{Tan}[d+ex])+2i\sqrt{a-ib-c}\sqrt{c+\operatorname{Tan}[d+ex]}(b+a\operatorname{Tan}[d+ex])\right)\right] \right) / \right. \\
& \left. \left( 8\sqrt{a-ib-c}c^3(-i+\operatorname{Tan}[d+ex]) \right) \right) + \sqrt{a-ib-c} \left( b\sqrt{a+ib-c}(5b^2-4c(3a+2c)) \right. \\
& \left. \operatorname{Log}\left[2c+b\operatorname{Tan}[d+ex]+2\sqrt{c}\sqrt{c+\operatorname{Tan}[d+ex]}(b+a\operatorname{Tan}[d+ex])\right] + 8c^{7/2} \operatorname{Log}\left[\left(2c+b(-i+\operatorname{Tan}[d+ex]) - \right. \right. \right. \\
& \left. \left. \left. 2i\left(a\operatorname{Tan}[d+ex]+\sqrt{a+ib-c}\sqrt{c+\operatorname{Tan}[d+ex]}(b+a\operatorname{Tan}[d+ex])\right)\right)\right] \right) / \left( 8\sqrt{a+ib-c}c^3(i+\operatorname{Tan}[d+ex]) \right) \right) \\
& \left( -\frac{5b^3\sqrt{-\frac{a}{-1+\cos[2(d+ex)]}-\frac{c}{-1+\cos[2(d+ex)]}+\frac{a\cos[2(d+ex)]}{-1+\cos[2(d+ex)]}-\frac{c\cos[2(d+ex)]}{-1+\cos[2(d+ex)]}-\frac{b\sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{8c^3(a+c-a\cos[2(d+ex)]+c\cos[2(d+ex)]+b\sin[2(d+ex)])} + \right.
\end{aligned}$$

$$3 a b \sqrt{\frac{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{2 c^2 (a+c-a \cos[2(d+e x)]+c \cos[2(d+e x)]+b \sin[2(d+e x)])}} +$$

$$b \sqrt{\frac{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{c (a+c-a \cos[2(d+e x)]+c \cos[2(d+e x)]+b \sin[2(d+e x)])}} +$$

$$\left. \frac{\sin[2(d+e x)] \sqrt{\frac{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{a+c-a \cos[2(d+e x)]+c \cos[2(d+e x)]+b \sin[2(d+e x)]}}}{\right\}$$

$$\left. \tan[d+e x] \sqrt{a+\cot[d+e x]^2(c+b \tan[d+e x])} \right\} /$$

$$\left( 16 \sqrt{a-i b-c} \sqrt{a+i b-c} c^{7/2} e \sqrt{c+\tan[d+e x]} (b+a \tan[d+e x]) \right.$$

$$\left. - \left( \left( \left( b \sqrt{a-i b-c} \sqrt{a+i b-c} (-5 b^2+4 c(3 a+2 c)) \log[\tan[d+e x]] - 8 \sqrt{a+i b-c} c^{7/2} \log\left[(-2 c-2 i a \tan[d+e x]-\right.\right.\right.\right.\right.$$

$$\left. \left. \left. b(i+\tan[d+e x])+2 i \sqrt{a-i b-c} \sqrt{c+\tan[d+e x]}(b+a \tan[d+e x])\right] \right) / \left( 8 \sqrt{a-i b-c} c^3(-i+\tan[d+e x]) \right) \right) \right) +$$

$$\sqrt{a-i b-c} \left( b \sqrt{a+i b-c} (5 b^2-4 c(3 a+2 c)) \log[2 c+b \tan[d+e x]+2 \sqrt{c} \sqrt{c+\tan[d+e x]}(b+a \tan[d+e x])] + \right.$$

$$\left. 8 c^{7/2} \log\left[2 c+b(-i+\tan[d+e x])-2 i\left(a \tan[d+e x]+\sqrt{a+i b-c} \sqrt{c+\tan[d+e x]}(b+a \tan[d+e x])\right)\right] \right) /$$

$$\left( 8 \sqrt{a+i b-c} c^3(i+\tan[d+e x]) \right) \right) \tan[d+e x] (a \sec[d+e x]^2 \tan[d+e x]+\sec[d+e x]^2(b+a \tan[d+e x]))$$

$$\sqrt{a+\cot[d+e x]^2(c+b \tan[d+e x])} \left. \right) / \left( 32 \sqrt{a-i b-c} \sqrt{a+i b-c} c^{7/2} (c+\tan[d+e x])(b+a \tan[d+e x])^{3/2} \right) +$$

$$\left( \left( b \sqrt{a-i b-c} \sqrt{a+i b-c} (-5 b^2+4 c(3 a+2 c)) \log[\tan[d+e x]] - 8 \sqrt{a+i b-c} c^{7/2} \log\left[(-2 c-2 i a \tan[d+e x]-\right.\right.\right.$$

$$\left. \left. \left. b(i+\tan[d+e x])+2 i \sqrt{a-i b-c} \sqrt{c+\tan[d+e x]}(b+a \tan[d+e x])\right] \right) / \left( 8 \sqrt{a-i b-c} c^3(-i+\tan[d+e x]) \right) \right) \right) +$$

$$\sqrt{a-i b-c} \left( b \sqrt{a+i b-c} (5 b^2-4 c(3 a+2 c)) \log[2 c+b \tan[d+e x]+2 \sqrt{c} \sqrt{c+\tan[d+e x]}(b+a \tan[d+e x])] + \right.$$





$$\begin{aligned}
& \left. \left( \frac{2i \sqrt{a-ib-c} \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex])}{\left( \sqrt{a-ib-c} c (-i+\tan[d+ex]) \right)} \right) + \right. \\
& \left. \sqrt{a-ib-c} \left( -b \sqrt{a+ib-c} \operatorname{Log}[2c+b \tan[d+ex]+2\sqrt{c} \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex])] + c^{3/2} \operatorname{Log}[2c+b(-i+\tan[d+ex])] - \right. \right. \\
& \left. \left. 2i \left( a \tan[d+ex] + \sqrt{a+ib-c} \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex]) \right) \right) \right) / \left( \sqrt{a+ib-c} c (i+\tan[d+ex]) \right) \Bigg) \\
& \left( \frac{b \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{c(a+c-a \cos[2(d+ex)]+c \cos[2(d+ex)]+b \sin[2(d+ex)])} - \right. \\
& \left. \frac{\sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{a+c-a \cos[2(d+ex)]+c \cos[2(d+ex)]+b \sin[2(d+ex)]} \right) \\
& \left. \tan[d+ex] \sqrt{a+\cot[d+ex]^2(c+b \tan[d+ex])} \right) / \\
& \left( 2 \sqrt{a-ib-c} \sqrt{a+ib-c} c^{3/2} e \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex]) \right. \\
& \left. \left( \left( b \sqrt{a-ib-c} \sqrt{a+ib-c} \operatorname{Log}[\tan[d+ex]] - \sqrt{a+ib-c} c^{3/2} \operatorname{Log}[-2c-2ia \tan[d+ex]-b(i+\tan[d+ex])] + \right. \right. \right. \\
& \left. \left. \frac{2i \sqrt{a-ib-c} \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex])}{\left( \sqrt{a-ib-c} c (-i+\tan[d+ex]) \right)} + \sqrt{a-ib-c} \right. \right. \\
& \left. \left. \left( -b \sqrt{a+ib-c} \operatorname{Log}[2c+b \tan[d+ex]+2\sqrt{c} \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex])] + c^{3/2} \operatorname{Log}[2c+b(-i+\tan[d+ex])] - \right. \right. \right. \\
& \left. \left. \left. 2i \left( a \tan[d+ex] + \sqrt{a+ib-c} \sqrt{c+\tan[d+ex]} (b+a \tan[d+ex]) \right) \right) \right) \right) / \left( \sqrt{a+ib-c} c (i+\tan[d+ex]) \right) \Bigg) \\
& \left. \tan[d+ex] \left( a \sec[d+ex]^2 \tan[d+ex] + \sec[d+ex]^2 (b+a \tan[d+ex]) \right) \sqrt{a+\cot[d+ex]^2(c+b \tan[d+ex])} \right) / \\
& \left( 4 \sqrt{a-ib-c} \sqrt{a+ib-c} c^{3/2} (c+\tan[d+ex]) (b+a \tan[d+ex])^{3/2} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2\sqrt{a-ib-c}\sqrt{a+ib-c}c^{3/2}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])} \\
& \left( b\sqrt{a-ib-c}\sqrt{a+ib-c}\log[\tan[d+ex]] - \sqrt{a+ib-c}c^{3/2}\log\left[(-2c-2ia\tan[d+ex]-b(i+\tan[d+ex]))+2\right. \right. \\
& \quad \left. \left. i\sqrt{a-ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])\right]/\left(\sqrt{a-ib-c}c(-i+\tan[d+ex])\right)\right] + \\
& \quad \sqrt{a-ib-c}\left[-b\sqrt{a+ib-c}\log[2c+b\tan[d+ex]+2\sqrt{c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])]+ \right. \\
& \quad \left. c^{3/2}\log\left[2c+b(-i+\tan[d+ex])-2i\left(a\tan[d+ex]+\sqrt{a+ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])\right)\right]/\right. \\
& \quad \left.\left(\sqrt{a+ib-c}c(i+\tan[d+ex])\right)\right]\right)\sec[d+ex]^2\sqrt{a+\cot[d+ex]^2(c+b\tan[d+ex])}- \\
& \left(\left(b\sqrt{a-ib-c}\sqrt{a+ib-c}\log[\tan[d+ex]] - \sqrt{a+ib-c}c^{3/2}\log\left[(-2c-2ia\tan[d+ex]-b(i+\tan[d+ex]))+2\right. \right. \right. \\
& \quad \left. \left. 2i\sqrt{a-ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])\right]/\left(\sqrt{a-ib-c}c(-i+\tan[d+ex])\right)\right] + \sqrt{a-ib-c} \right. \\
& \quad \left. \left(-b\sqrt{a+ib-c}\log[2c+b\tan[d+ex]+2\sqrt{c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])]+c^{3/2}\log\left[2c+b(-i+\tan[d+ex])-\right. \right. \right. \\
& \quad \left. \left. 2i\left(a\tan[d+ex]+\sqrt{a+ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])\right)\right]/\left(\sqrt{a+ib-c}c(i+\tan[d+ex])\right)\right]\right)\right) \\
& \tan[d+ex](b\csc[d+ex]^2-2\cot[d+ex]\csc[d+ex]^2(c+b\tan[d+ex]))\left/\left(4\sqrt{a-ib-c}\sqrt{a+ib-c}c^{3/2} \right. \right. \\
& \quad \left. \left.\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])\sqrt{a+\cot[d+ex]^2(c+b\tan[d+ex])}\right)- \right. \\
& \frac{1}{2\sqrt{a-ib-c}\sqrt{a+ib-c}c^{3/2}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])} \tan[d+ex]\sqrt{a+\cot[d+ex]^2(c+b\tan[d+ex])} \\
& \left(b\sqrt{a-ib-c}\sqrt{a+ib-c}\csc[d+ex]\sec[d+ex]-\sqrt{a-ib-c}\sqrt{a+ib-c}c^{5/2}(-i+\tan[d+ex])\right) \\
& \left(\frac{-2ia\sec[d+ex]^2-b\sec[d+ex]^2+\frac{i\sqrt{a-ib-c}(a\sec[d+ex]^2\tan[d+ex]+\sec[d+ex]^2(b+a\tan[d+ex]))}{\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])}}{\sqrt{a-ib-c}c(-i+\tan[d+ex])}- \right. \\
& \quad \left. \left(\sec[d+ex]^2(-2c-2ia\tan[d+ex]-b(i+\tan[d+ex]))+2i\sqrt{a-ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])\right)\right)\left/ \right.
\end{aligned}$$





Result (type 3, 2104 leaves):

$$\begin{aligned}
 & - \left( \left( \left( \sqrt{a - i b - c} \operatorname{Log} \left[ \frac{2 \left( \frac{b(-i + \operatorname{Tan}[d + e x]) + 2(c - i a \operatorname{Tan}[d + e x])}{\sqrt{a + i b - c}} - 2 i \sqrt{c + \operatorname{Tan}[d + e x]} (b + a \operatorname{Tan}[d + e x]) \right)}{i + \operatorname{Tan}[d + e x]} \right] \right) - \right. \\
 & \quad \left. \sqrt{a + i b - c} \operatorname{Log} \left[ \frac{2 \left( -\frac{b(i + \operatorname{Tan}[d + e x]) + 2(c + i a \operatorname{Tan}[d + e x])}{\sqrt{a - i b - c}} + 2 i \sqrt{c + \operatorname{Tan}[d + e x]} (b + a \operatorname{Tan}[d + e x]) \right)}{-i + \operatorname{Tan}[d + e x]} \right] \right) \operatorname{Sin}[2(d + e x)] \\
 & \quad \sqrt{\left( -\frac{a}{-1 + \operatorname{Cos}[2(d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2(d + e x)]} + \frac{a \operatorname{Cos}[2(d + e x)]}{-1 + \operatorname{Cos}[2(d + e x)]} - \frac{c \operatorname{Cos}[2(d + e x)]}{-1 + \operatorname{Cos}[2(d + e x)]} - \frac{b \operatorname{Sin}[2(d + e x)]}{-1 + \operatorname{Cos}[2(d + e x)]} \right)} \\
 & \quad \left. \operatorname{Tan}[d + e x] \sqrt{a + \operatorname{Cot}[d + e x]^2 (c + b \operatorname{Tan}[d + e x])} \right) / \\
 & \left( 2 \sqrt{a - i b - c} \sqrt{a + i b - c} e^{(-a - c + (a - c) \operatorname{Cos}[2(d + e x)] - b \operatorname{Sin}[2(d + e x)])} \sqrt{c + \operatorname{Tan}[d + e x]} (b + a \operatorname{Tan}[d + e x]) \right. \\
 & \left. - \frac{1}{4 \sqrt{a - i b - c} \sqrt{a + i b - c} (c + \operatorname{Tan}[d + e x] (b + a \operatorname{Tan}[d + e x]))^{3/2}} \right) \\
 & \left( \left( \sqrt{a - i b - c} \operatorname{Log} \left[ \frac{2 \left( \frac{b(-i + \operatorname{Tan}[d + e x]) + 2(c - i a \operatorname{Tan}[d + e x])}{\sqrt{a + i b - c}} - 2 i \sqrt{c + \operatorname{Tan}[d + e x]} (b + a \operatorname{Tan}[d + e x]) \right)}{i + \operatorname{Tan}[d + e x]} \right] \right) - \right. \\
 & \quad \left. \sqrt{a + i b - c} \operatorname{Log} \left[ \frac{2 \left( -\frac{b(i + \operatorname{Tan}[d + e x]) + 2(c + i a \operatorname{Tan}[d + e x])}{\sqrt{a - i b - c}} + 2 i \sqrt{c + \operatorname{Tan}[d + e x]} (b + a \operatorname{Tan}[d + e x]) \right)}{-i + \operatorname{Tan}[d + e x]} \right] \right) \\
 & \quad \operatorname{Tan}[d + e x] (a \operatorname{Sec}[d + e x]^2 \operatorname{Tan}[d + e x] + \operatorname{Sec}[d + e x]^2 (b + a \operatorname{Tan}[d + e x])) \sqrt{a + \operatorname{Cot}[d + e x]^2 (c + b \operatorname{Tan}[d + e x])} + \\
 & \quad \left( \left( \sqrt{a - i b - c} \operatorname{Log} \left[ \frac{2 \left( \frac{b(-i + \operatorname{Tan}[d + e x]) + 2(c - i a \operatorname{Tan}[d + e x])}{\sqrt{a + i b - c}} - 2 i \sqrt{c + \operatorname{Tan}[d + e x]} (b + a \operatorname{Tan}[d + e x]) \right)}{i + \operatorname{Tan}[d + e x]} \right] \right) - \right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{a+ib-c} \operatorname{Log} \left[ \frac{2 \left( -\frac{b(i+\tan[d+ex])+2(c+ia \tan[d+ex])}{\sqrt{a-ib-c}} + 2i \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \right)}{-i+\tan[d+ex]} \right] \operatorname{Sec}[d+ex]^2 \right. \\
& \left. \sqrt{a+\cot[d+ex]^2(c+b \tan[d+ex])} \right] / \left( 2 \sqrt{a-ib-c} \sqrt{a+ib-c} \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \right) + \\
& \left( \left( \sqrt{a-ib-c} \operatorname{Log} \left[ \frac{2 \left( \frac{b(-i+\tan[d+ex])+2(c-ia \tan[d+ex])}{\sqrt{a+ib-c}} - 2i \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \right)}{i+\tan[d+ex]} \right] - \right. \right. \\
& \left. \left. \sqrt{a+ib-c} \operatorname{Log} \left[ \frac{2 \left( -\frac{b(i+\tan[d+ex])+2(c+ia \tan[d+ex])}{\sqrt{a-ib-c}} + 2i \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \right)}{-i+\tan[d+ex]} \right] \right) \right) \\
& \left. \operatorname{Tan}[d+ex] \left( b \operatorname{Csc}[d+ex]^2 - 2 \operatorname{Cot}[d+ex] \operatorname{Csc}[d+ex]^2 (c+b \tan[d+ex]) \right) \right) / \\
& \left( 4 \sqrt{a-ib-c} \sqrt{a+ib-c} \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \sqrt{a+\cot[d+ex]^2(c+b \tan[d+ex])} \right) + \\
& \frac{1}{2 \sqrt{a-ib-c} \sqrt{a+ib-c} \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex])} \operatorname{Tan}[d+ex] \sqrt{a+\cot[d+ex]^2(c+b \tan[d+ex])} \\
& \left( \left( \sqrt{a-ib-c} (i+\tan[d+ex]) \left( \frac{2 \left( \frac{-2ia \operatorname{Sec}[d+ex]^2 + b \operatorname{Sec}[d+ex]^2}{\sqrt{a+ib-c}} - \frac{i(a \operatorname{Sec}[d+ex]^2 \operatorname{Tan}[d+ex] + \operatorname{Sec}[d+ex]^2 (b+a \tan[d+ex]))}{\sqrt{c+\tan[d+ex]}(b+a \tan[d+ex])} \right)}{i+\tan[d+ex]} - \frac{1}{(i+\tan[d+ex])^2} \right) \right) \right) \\
& \left. 2 \operatorname{Sec}[d+ex]^2 \left( \frac{b(-i+\tan[d+ex])+2(c-ia \tan[d+ex])}{\sqrt{a+ib-c}} - 2i \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \right) \right) / \\
& \left( 2 \left( \frac{b(-i+\tan[d+ex])+2(c-ia \tan[d+ex])}{\sqrt{a+ib-c}} - 2i \sqrt{c+\tan[d+ex]}(b+a \tan[d+ex]) \right) \right) -
\end{aligned}$$

$$\left( \sqrt{a + i b - c} (-i + \tan[d + e x]) \left( \frac{2 \left( -\frac{2 i a \sec[d + e x]^2 + b \sec[d + e x]^2}{\sqrt{a - i b - c}} + \frac{i (a \sec[d + e x]^2 \tan[d + e x] + \sec[d + e x]^2 (b + a \tan[d + e x]))}{\sqrt{c + \tan[d + e x] (b + a \tan[d + e x])}} \right)}{-i + \tan[d + e x]} - \frac{1}{(-i + \tan[d + e x])^2} \right. \right. \\ \left. \left. 2 \sec[d + e x]^2 \left( -\frac{b (i + \tan[d + e x]) + 2 (c + i a \tan[d + e x])}{\sqrt{a - i b - c}} + 2 i \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right) \right) \right) / \\ \left( 2 \left( -\frac{b (i + \tan[d + e x]) + 2 (c + i a \tan[d + e x])}{\sqrt{a - i b - c}} + 2 i \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right) \right) \right)$$

**Problem 4: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[d + e x]}{\sqrt{a + b \cot[d + e x] + c \cot[d + e x]^2}} dx$$

Optimal (type 3, 349 leaves, 10 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2 a + b \cot[d + e x]}{2 \sqrt{a} \sqrt{a + b \cot[d + e x] + c \cot[d + e x]^2}}\right]}{\sqrt{a} e} + \frac{\sqrt{a - c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \operatorname{ArcTanh}\left[\frac{a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} + b \cot[d + e x]}{\sqrt{2} \sqrt{a - c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a + b \cot[d + e x] + c \cot[d + e x]^2}}\right]}{\sqrt{2} \sqrt{a^2 + b^2 - 2 a c + c^2} e} - \\ \frac{\sqrt{a - c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \operatorname{ArcTanh}\left[\frac{a - c + \sqrt{a^2 + b^2 - 2 a c + c^2} + b \cot[d + e x]}{\sqrt{2} \sqrt{a - c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a + b \cot[d + e x] + c \cot[d + e x]^2}}\right]}{\sqrt{2} \sqrt{a^2 + b^2 - 2 a c + c^2} e}$$

Result (type 4, 64 621 leaves): Display of huge result suppressed!

**Problem 5: Humongous result has more than 200000 leaves.**

$$\int \frac{\tan[d + e x]^3}{\sqrt{a + b \cot[d + e x] + c \cot[d + e x]^2}} dx$$

Optimal (type 3, 501 leaves, 14 steps):

$$\begin{aligned}
& - \frac{\text{ArcTanh}\left[\frac{2a+b\text{Cot}[d+ex]}{2\sqrt{a}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{\sqrt{a}e} + \frac{(3b^2-4ac)\text{ArcTanh}\left[\frac{2a+b\text{Cot}[d+ex]}{2\sqrt{a}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{8a^{5/2}e} \\
& + \frac{\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\text{ArcTanh}\left[\frac{a-c-\sqrt{a^2+b^2-2ac+c^2}+b\text{Cot}[d+ex]}{\sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} \\
& - \frac{\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\text{ArcTanh}\left[\frac{a-c+\sqrt{a^2+b^2-2ac+c^2}+b\text{Cot}[d+ex]}{\sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{\sqrt{2}\sqrt{a^2+b^2-2ac+c^2}e} \\
& + \frac{3b\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}\text{Tan}[d+ex]}{4a^2e} + \frac{\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}\text{Tan}[d+ex]^2}{2ae}
\end{aligned}$$

Result (type ?, 325525 leaves): Display of huge result suppressed!

**Problem 6: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cot}[d+ex]^5 \sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2} dx$$

Optimal (type 3, 976 leaves, 21 steps):

$$\begin{aligned}
& - \left( \left( \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \right. \\
& \quad \left. \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) - \frac{b \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{2\sqrt{c} e} + \frac{b (b^2 - 4ac) \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{16c^{5/2} e} - \\
& \quad \frac{b (7b^2 - 12ac) (b^2 - 4ac) \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{256c^{9/2} e} + \\
& \quad \left( \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) - \frac{\sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}}{e} - \frac{b (b + 2c \cot [d + ex]) \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}}{8c^2 e} + \\
& \quad \frac{b (7b^2 - 12ac) (b + 2c \cot [d + ex]) \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}}{128c^4 e} + \\
& \quad \frac{(a + b \cot [d + ex] + c \cot [d + ex]^2)^{3/2}}{3ce} - \\
& \quad \frac{\cot [d + ex]^2 (a + b \cot [d + ex] + c \cot [d + ex]^2)^{3/2}}{5ce} - \\
& \quad \frac{(35b^2 - 32ac - 42bc \cot [d + ex]) (a + b \cot [d + ex] + c \cot [d + ex]^2)^{3/2}}{240c^3 e}
\end{aligned}$$

Result (type 3, 4237 leaves):

$$\begin{aligned}
& \frac{1}{e} \left( - \frac{-105b^4 + 460ab^2c - 256a^2c^2 + 296b^2c^2 - 768ac^3 + 2944c^4}{1920c^4} + \frac{(-35b^3 \cos [d + ex] + 116abc \cos [d + ex] + 104b^2c^2 \cos [d + ex]) \csc [d + ex]}{960c^3} \right. \\
& \quad \left. \frac{(7b^2 - 16ac + 176c^2) \csc [d + ex]^2}{240c^2} - \frac{b \cot [d + ex] \csc [d + ex]^2}{40c} - \frac{1}{5} \csc [d + ex]^4 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)]}{-1 + \cos[2(d+ex)]}} + \\
& \left( b(7b^4 - 8b^2c(5a+2c) + 16c^2(3a^2 + 4ac + 8c^2)) \log[\tan[d+ex]] - 128\sqrt{a-ib-c} c^{9/2} \log\left[(-2c - 2ia \tan[d+ex] - \right. \right. \\
& \quad \left. \left. b(i + \tan[d+ex]) + 2i\sqrt{a-ib-c} \sqrt{c + \tan[d+ex]}(b + a \tan[d+ex])\right]\right) / \left(128(a-ib-c)^{3/2} c^4 (-i + \tan[d+ex])\right) - \\
& b(7b^4 - 8b^2c(5a+2c) + 16c^2(3a^2 + 4ac + 8c^2)) \log\left[2c + b \tan[d+ex] + 2\sqrt{c} \sqrt{c + \tan[d+ex]}(b + a \tan[d+ex])\right] + \\
& 128\sqrt{a+ib-c} c^{9/2} \log\left[2c + b(-i + \tan[d+ex]) - 2i\left(a \tan[d+ex] + \sqrt{a+ib-c} \sqrt{c + \tan[d+ex]}(b + a \tan[d+ex])\right)\right] / \\
& \left(128(a+ib-c)^{3/2} c^4 (i + \tan[d+ex])\right) \left( \frac{7b^5 \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{128c^4(a+c - a \cos[2(d+ex)] + c \cos[2(d+ex)] + b \sin[2(d+ex)])} - \right. \\
& \frac{5ab^3 \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{16c^3(a+c - a \cos[2(d+ex)] + c \cos[2(d+ex)] + b \sin[2(d+ex)])} + \\
& \frac{3a^2b \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{8c^2(a+c - a \cos[2(d+ex)] + c \cos[2(d+ex)] + b \sin[2(d+ex)])} - \\
& \frac{b^3 \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{8c^2(a+c - a \cos[2(d+ex)] + c \cos[2(d+ex)] + b \sin[2(d+ex)])} + \\
& \frac{ab \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{2c(a+c - a \cos[2(d+ex)] + c \cos[2(d+ex)] + b \sin[2(d+ex)])} + \\
& \frac{b \cos[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{a+c - a \cos[2(d+ex)] + c \cos[2(d+ex)] + b \sin[2(d+ex)]} +
\end{aligned}$$

$$\begin{aligned}
& \frac{a \operatorname{Sin}[2(d+ex)] \sqrt{-\frac{a}{-1+\operatorname{Cos}[2(d+ex)]} - \frac{c}{-1+\operatorname{Cos}[2(d+ex)]} + \frac{a \operatorname{Cos}[2(d+ex)]}{-1+\operatorname{Cos}[2(d+ex)]} - \frac{c \operatorname{Cos}[2(d+ex)]}{-1+\operatorname{Cos}[2(d+ex)]} - \frac{b \operatorname{Sin}[2(d+ex)]}{-1+\operatorname{Cos}[2(d+ex)]}}}{a+c-a \operatorname{Cos}[2(d+ex)]+c \operatorname{Cos}[2(d+ex)]+b \operatorname{Sin}[2(d+ex)]} - \\
& \left. \frac{c \operatorname{Sin}[2(d+ex)] \sqrt{-\frac{a}{-1+\operatorname{Cos}[2(d+ex)]} - \frac{c}{-1+\operatorname{Cos}[2(d+ex)]} + \frac{a \operatorname{Cos}[2(d+ex)]}{-1+\operatorname{Cos}[2(d+ex)]} - \frac{c \operatorname{Cos}[2(d+ex)]}{-1+\operatorname{Cos}[2(d+ex)]} - \frac{b \operatorname{Sin}[2(d+ex)]}{-1+\operatorname{Cos}[2(d+ex)]}}}{a+c-a \operatorname{Cos}[2(d+ex)]+c \operatorname{Cos}[2(d+ex)]+b \operatorname{Sin}[2(d+ex)]} \right) \\
& \left. \operatorname{Tan}[d+ex] \sqrt{a+\operatorname{Cot}[d+ex]^2(c+b \operatorname{Tan}[d+ex])} \right) \Bigg/ \left( 256 c^{9/2} e \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \\
& \left( -\frac{1}{512 c^{9/2} (c+\operatorname{Tan}[d+ex] (b+a \operatorname{Tan}[d+ex]))^{3/2}} \left( b(7b^4-8b^2c(5a+2c)+16c^2(3a^2+4ac+8c^2)) \operatorname{Log}[\operatorname{Tan}[d+ex]] - \right. \right. \\
& \quad \left. \left. 128 \sqrt{a-ib-c} c^{9/2} \operatorname{Log} \left[ \left( -2c-2ia \operatorname{Tan}[d+ex] - b(i+\operatorname{Tan}[d+ex]) + 2i \sqrt{a-ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \right] \right) \Bigg/ \right. \\
& \quad \left. \left( 128 (a-ib-c)^{3/2} c^4 (-i+\operatorname{Tan}[d+ex]) \right) \right) - b(7b^4-8b^2c(5a+2c)+16c^2(3a^2+4ac+8c^2)) \\
& \quad \left. \operatorname{Log} \left[ 2c+b \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right] + 128 \sqrt{a+ib-c} c^{9/2} \operatorname{Log} \left[ \left( 2c+b(-i+\operatorname{Tan}[d+ex]) - \right. \right. \right. \\
& \quad \left. \left. 2i \left( a \operatorname{Tan}[d+ex] + \sqrt{a+ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \right) \right] \Bigg/ \left( 128 (a+ib-c)^{3/2} c^4 (i+\operatorname{Tan}[d+ex]) \right) \Bigg) \\
& \quad \left. \operatorname{Tan}[d+ex] \left( a \operatorname{Sec}[d+ex]^2 \operatorname{Tan}[d+ex] + \operatorname{Sec}[d+ex]^2 (b+a \operatorname{Tan}[d+ex]) \right) \sqrt{a+\operatorname{Cot}[d+ex]^2(c+b \operatorname{Tan}[d+ex])} + \right. \\
& \quad \left. \frac{1}{256 c^{9/2} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex])} \left( b(7b^4-8b^2c(5a+2c)+16c^2(3a^2+4ac+8c^2)) \operatorname{Log}[\operatorname{Tan}[d+ex]] - \right. \right. \\
& \quad \left. \left. 128 \sqrt{a-ib-c} c^{9/2} \operatorname{Log} \left[ \left( -2c-2ia \operatorname{Tan}[d+ex] - b(i+\operatorname{Tan}[d+ex]) + 2i \sqrt{a-ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \right] \right) \Bigg/ \right. \\
& \quad \left. \left( 128 (a-ib-c)^{3/2} c^4 (-i+\operatorname{Tan}[d+ex]) \right) \right) - \\
& \quad \left. b(7b^4-8b^2c(5a+2c)+16c^2(3a^2+4ac+8c^2)) \operatorname{Log} \left[ 2c+b \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right] + \right. \\
& \quad \left. 128 \sqrt{a+ib-c} c^{9/2} \operatorname{Log} \left[ \left( 2c+b(-i+\operatorname{Tan}[d+ex]) - 2i \left( a \operatorname{Tan}[d+ex] + \sqrt{a+ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \right) \right] \Bigg/ \right. \\
& \quad \left. \left( 128 (a+ib-c)^{3/2} c^4 (i+\operatorname{Tan}[d+ex]) \right) \right) \operatorname{Sec}[d+ex]^2 \sqrt{a+\operatorname{Cot}[d+ex]^2(c+b \operatorname{Tan}[d+ex])} + \\
& \quad \left( \left( b(7b^4-8b^2c(5a+2c)+16c^2(3a^2+4ac+8c^2)) \operatorname{Log}[\operatorname{Tan}[d+ex]] - 128 \sqrt{a-ib-c} c^{9/2} \operatorname{Log} \left[ \left( -2c-2ia \operatorname{Tan}[d+ex] - \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( \frac{b (\mathfrak{i} + \tan [d + e x]) + 2 \mathfrak{i} \sqrt{a - \mathfrak{i} b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}}{(128 (a - \mathfrak{i} b - c)^{3/2} c^4 (-\mathfrak{i} + \tan [d + e x]))} \right) - \right. \\
& \left. b (7 b^4 - 8 b^2 c (5 a + 2 c) + 16 c^2 (3 a^2 + 4 a c + 8 c^2)) \log [2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}] + \right. \\
& \left. 128 \sqrt{a + \mathfrak{i} b - c} c^{9/2} \log \left[ \left( 2 c + b (-\mathfrak{i} + \tan [d + e x]) - 2 \mathfrak{i} \left( a \tan [d + e x] + \sqrt{a + \mathfrak{i} b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right] \right) / \\
& \left( 128 (a + \mathfrak{i} b - c)^{3/2} c^4 (\mathfrak{i} + \tan [d + e x]) \right) \tan [d + e x] (b \csc [d + e x]^2 - 2 \cot [d + e x] \csc [d + e x]^2 (c + b \tan [d + e x])) / \\
& \left( 512 c^{9/2} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \sqrt{a + \cot [d + e x]^2 (c + b \tan [d + e x])} \right) + \\
& \frac{1}{256 c^{9/2} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \tan [d + e x] \sqrt{a + \cot [d + e x]^2 (c + b \tan [d + e x])} \\
& \left( b (7 b^4 - 8 b^2 c (5 a + 2 c) + 16 c^2 (3 a^2 + 4 a c + 8 c^2)) \csc [d + e x] \sec [d + e x] - \left( b (7 b^4 - 8 b^2 c (5 a + 2 c) + 16 c^2 (3 a^2 + 4 a c + 8 c^2)) \right. \right. \\
& \left. \left. \left( b \sec [d + e x]^2 + \frac{\sqrt{c} (a \sec [d + e x]^2 \tan [d + e x] + \sec [d + e x]^2 (b + a \tan [d + e x]))}{\sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right) \right) \right) / \\
& \left( 2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) - \left( 16384 (a - \mathfrak{i} b - c)^2 c^{17/2} (-\mathfrak{i} + \tan [d + e x]) \right. \\
& \left. \frac{-2 \mathfrak{i} a \sec [d + e x]^2 - b \sec [d + e x]^2 + \frac{\mathfrak{i} \sqrt{a - \mathfrak{i} b - c} (a \sec [d + e x]^2 \tan [d + e x] + \sec [d + e x]^2 (b + a \tan [d + e x]))}{\sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}}}{128 (a - \mathfrak{i} b - c)^{3/2} c^4 (-\mathfrak{i} + \tan [d + e x])} \right. \\
& \left. \left( \sec [d + e x]^2 (-2 c - 2 \mathfrak{i} a \tan [d + e x] - b (\mathfrak{i} + \tan [d + e x]) + 2 \mathfrak{i} \sqrt{a - \mathfrak{i} b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}) \right) \right) / \\
& \left( 128 (a - \mathfrak{i} b - c)^{3/2} c^4 (-\mathfrak{i} + \tan [d + e x])^2 \right) \right) / \\
& \left( -2 c - 2 \mathfrak{i} a \tan [d + e x] - b (\mathfrak{i} + \tan [d + e x]) + 2 \mathfrak{i} \sqrt{a - \mathfrak{i} b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) +
\end{aligned}$$





$$\begin{aligned}
& \left( \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) + \frac{b \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{2\sqrt{c} e} - \frac{b (b^2 - 4ac) \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{16c^{5/2} e} - \\
& \quad \left( \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) + \frac{\sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}}{e} + \frac{b (b + 2c \cot [d + ex]) \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}}{8c^2 e} - \\
& \quad \frac{(a + b \cot [d + ex] + c \cot [d + ex]^2)^{3/2}}{3ce}
\end{aligned}$$

Result (type 3, 3416 leaves):

$$\begin{aligned}
& \frac{\left( \frac{3b^2 - 8ac + 32c^2}{24c^2} - \frac{b \cot [d + ex]}{12c} - \frac{1}{3} \text{Csc} [d + ex]^2 \right) \sqrt{\frac{-a - c + a \cos [2 (d + ex)] - c \cos [2 (d + ex)] - b \sin [2 (d + ex)]}{-1 + \cos [2 (d + ex)]}}}{e} + \\
& \left( b (b^2 - 4c (a + 2c)) \log [\tan [d + ex]] - 8 \sqrt{a + ib - c} c^{5/2} \right. \\
& \quad \left. \log \left[ \left( i \left( b + 2ic + 2a \tan [d + ex] + ib \tan [d + ex] + 2 \sqrt{a + ib - c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right) \right) \right] \right) / \\
& \quad \left( 8 (a + ib - c)^{3/2} c^2 (i + \tan [d + ex]) \right) - b (b^2 - 4c (a + 2c)) \log \left[ 2c + b \tan [d + ex] + 2\sqrt{c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right] + \\
& \quad 8 \sqrt{a - ib - c} c^{5/2} \log \left[ \left( b (i + \tan [d + ex]) + 2 \left( c + ia \tan [d + ex] - i \sqrt{a - ib - c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right) \right) \right] /
\end{aligned}$$

$$\left( 8 (a - i b - c)^{3/2} c^2 (-i + \tan[d + e x]) \right) \left( \frac{b^3 \sqrt{-\frac{a}{-1 + \cos[2(d + e x)]} - \frac{c}{-1 + \cos[2(d + e x)]} + \frac{a \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{c \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{b \sin[2(d + e x)]}{-1 + \cos[2(d + e x)]}}{8 c^2 (a + c - a \cos[2(d + e x)] + c \cos[2(d + e x)] + b \sin[2(d + e x)])} \right)$$

$$a b \frac{\sqrt{-\frac{a}{-1 + \cos[2(d + e x)]} - \frac{c}{-1 + \cos[2(d + e x)]} + \frac{a \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{c \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{b \sin[2(d + e x)]}{-1 + \cos[2(d + e x)]}}{2 c (a + c - a \cos[2(d + e x)] + c \cos[2(d + e x)] + b \sin[2(d + e x)])}$$

$$b \cos[2(d + e x)] \frac{\sqrt{-\frac{a}{-1 + \cos[2(d + e x)]} - \frac{c}{-1 + \cos[2(d + e x)]} + \frac{a \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{c \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{b \sin[2(d + e x)]}{-1 + \cos[2(d + e x)]}}{a + c - a \cos[2(d + e x)] + c \cos[2(d + e x)] + b \sin[2(d + e x)]}$$

$$a \sin[2(d + e x)] \frac{\sqrt{-\frac{a}{-1 + \cos[2(d + e x)]} - \frac{c}{-1 + \cos[2(d + e x)]} + \frac{a \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{c \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{b \sin[2(d + e x)]}{-1 + \cos[2(d + e x)]}}{a + c - a \cos[2(d + e x)] + c \cos[2(d + e x)] + b \sin[2(d + e x)]} +$$

$$c \sin[2(d + e x)] \frac{\sqrt{-\frac{a}{-1 + \cos[2(d + e x)]} - \frac{c}{-1 + \cos[2(d + e x)]} + \frac{a \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{c \cos[2(d + e x)]}{-1 + \cos[2(d + e x)]} - \frac{b \sin[2(d + e x)]}{-1 + \cos[2(d + e x)]}}{a + c - a \cos[2(d + e x)] + c \cos[2(d + e x)] + b \sin[2(d + e x)]}$$

$$\left. \tan[d + e x] \sqrt{a + \cot[d + e x]^2 (c + b \tan[d + e x])} \right/$$

$$\left( 16 c^{5/2} e \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \left( -\frac{1}{32 c^{5/2} (c + \tan[d + e x] (b + a \tan[d + e x]))^{3/2}} \right) \right)$$

$$\left( b (b^2 - 4 c (a + 2 c)) \operatorname{Log}[\tan[d + e x]] - 8 \sqrt{a + i b - c} c^{5/2} \operatorname{Log} \left[ \left( i \left( b + 2 i c + 2 a \tan[d + e x] + i b \tan[d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right) \right) / \left( 8 (a + i b - c)^{3/2} c^2 (i + \tan[d + e x]) \right) \right] - b (b^2 - 4 c (a + 2 c)) \right)$$

$$\operatorname{Log} \left[ 2 c + b \tan[d + e x] + 2 \sqrt{c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right] + 8 \sqrt{a - i b - c} c^{5/2} \operatorname{Log} \left[ \left( b (i + \tan[d + e x]) + 2 \left( c + i a \tan[d + e x] - i \sqrt{a - i b - c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right) \right) / \left( 8 (a - i b - c)^{3/2} c^2 (-i + \tan[d + e x]) \right) \right]$$

$$\left. \left( 2 \left( c + i a \tan[d + e x] - i \sqrt{a - i b - c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right) \right) / \left( 8 (a - i b - c)^{3/2} c^2 (-i + \tan[d + e x]) \right) \right]$$



$$\begin{aligned}
& \left( i b \tan [d+e x] + 2 \sqrt{a+i b-c} \sqrt{c+\tan [d+e x] (b+a \tan [d+e x])} \right) + \left( 64 (a-i b-c)^2 c^{9/2} (-i+\tan [d+e x]) \right. \\
& \left. \left( \left( b \sec [d+e x]^2 + 2 \left( i a \sec [d+e x]^2 - \frac{i \sqrt{a-i b-c} (a \sec [d+e x]^2 \tan [d+e x] + \sec [d+e x]^2 (b+a \tan [d+e x]))}{2 \sqrt{c+\tan [d+e x] (b+a \tan [d+e x])}} \right) \right) \right) / \right. \\
& \left. \left( 8 (a-i b-c)^{3/2} c^2 (-i+\tan [d+e x]) \right) - \left( \sec [d+e x]^2 (b (i+\tan [d+e x]) + 2 (c+i a \tan [d+e x] - \right. \right. \\
& \left. \left. i \sqrt{a-i b-c} \sqrt{c+\tan [d+e x] (b+a \tan [d+e x])} \right) \right) \right) / \left( 8 (a-i b-c)^{3/2} c^2 (-i+\tan [d+e x])^2 \right) \left. \right) / \left. \right) \\
& \left. \left( b (i+\tan [d+e x]) + 2 (c+i a \tan [d+e x] - i \sqrt{a-i b-c} \sqrt{c+\tan [d+e x] (b+a \tan [d+e x])}) \right) \right) \left. \right) \left. \right)
\end{aligned}$$

**Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot [d+e x] \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2} dx$$

Optimal (type 3, 602 leaves, 10 steps):

$$\begin{aligned}
& - \left( \left( \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \right. \\
& \quad \left. \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \right. \\
& \quad \left. \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) \right) / \\
& \quad \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} e \right) - \frac{b \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{2\sqrt{c} e} + \\
& \left( \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) / \\
& \quad \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} e \right) - \frac{\sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}}{e}
\end{aligned}$$

Result (type 3, 2871 leaves):

$$\begin{aligned}
& - \frac{\sqrt{\frac{-a - c + a \cos [2 (d + ex)] - c \cos [2 (d + ex)] - b \sin [2 (d + ex)]}{-1 + \cos [2 (d + ex)]}}}{e} - \left( \left( - \frac{b \log [\tan [d + ex]]}{\sqrt{c}} - \right. \right. \\
& \quad \left. \left. \sqrt{a + i b - c} \log \left[ \left( i \left( b + 2 i c + 2 a \tan [d + ex] + i b \tan [d + ex] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right) \right) \right] \right) / \right. \\
& \quad \left. \left( (a + i b - c)^{3/2} (i + \tan [d + ex]) \right) \right) + \frac{b \log [2 c + b \tan [d + ex] + 2 \sqrt{c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])}]}{\sqrt{c}} + \\
& \quad \left. \sqrt{a - i b - c} \log \left[ \left( b \left( i + \tan [d + ex] \right) + 2 \left( c + i a \tan [d + ex] - i \sqrt{a - i b - c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right) \right) \right] \right) / \right. \\
& \quad \left. \left( (a - i b - c)^{3/2} (-i + \tan [d + ex]) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( - \left( \left( b \cos[2(d+ex)] \right) \sqrt{\left( -\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]} \right)} \right) \right) / \left( -a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)] \right) - \\
& \frac{a \sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)]} + \\
& \frac{c \sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)]} \Big) \\
& \tan[d+ex] \sqrt{a + \cot[d+ex]^2 (c + b \tan[d+ex])} \Big) / \left( 2e \sqrt{c + \tan[d+ex]} (b + a \tan[d+ex]) \right) \\
& \left( \frac{1}{4 (c + \tan[d+ex] (b + a \tan[d+ex]))^{3/2}} \left( -\frac{b \log[\tan[d+ex]]}{\sqrt{c}} - \right. \right. \\
& \left. \left. \frac{\sqrt{a + ib - c} \log\left[ \left( i \left( b + 2ic + 2a \tan[d+ex] + ib \tan[d+ex] + 2\sqrt{a + ib - c} \sqrt{c + \tan[d+ex]} (b + a \tan[d+ex]) \right) \right)}{\left( (a + ib - c)^{3/2} (i + \tan[d+ex]) \right)} \right) + \frac{b \log\left[ 2c + b \tan[d+ex] + 2\sqrt{c} \sqrt{c + \tan[d+ex]} (b + a \tan[d+ex]) \right]}{\sqrt{c}} + \right. \\
& \left. \frac{\sqrt{a - ib - c} \log\left[ \left( b \left( i + \tan[d+ex] \right) + 2 \left( c + ia \tan[d+ex] - i\sqrt{a - ib - c} \sqrt{c + \tan[d+ex]} (b + a \tan[d+ex]) \right) \right)}{\left( (a - ib - c)^{3/2} (-i + \tan[d+ex]) \right)} \right] \right) \tan[d+ex] (a \sec[d+ex]^2 \tan[d+ex] + \sec[d+ex]^2 (b + a \tan[d+ex]))
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a + \cot [d + e x]^2 (c + b \tan [d + e x])} - \frac{1}{2 \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \left( -\frac{b \operatorname{Log}[\tan [d + e x]]}{\sqrt{c}} - \right. \\
& \left. \sqrt{a + i b - c} \operatorname{Log} \left[ \left( i \left( b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right] / \right. \\
& \left. \left( (a + i b - c)^{3/2} (i + \tan [d + e x]) \right) \right] + \frac{b \operatorname{Log} \left[ 2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right]}{\sqrt{c}} + \\
& \sqrt{a - i b - c} \operatorname{Log} \left[ \left( b (i + \tan [d + e x]) + 2 \left( c + i a \tan [d + e x] - i \sqrt{a - i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right] / \right. \\
& \left. \left( (a - i b - c)^{3/2} (-i + \tan [d + e x]) \right) \right] \left. \operatorname{Sec}[d + e x]^2 \sqrt{a + \cot [d + e x]^2 (c + b \tan [d + e x])} - \left( \left( -\frac{b \operatorname{Log}[\tan [d + e x]]}{\sqrt{c}} - \right. \right. \right. \\
& \left. \left. \sqrt{a + i b - c} \operatorname{Log} \left[ \left( i \left( b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right] / \right. \right. \\
& \left. \left. \left( (a + i b - c)^{3/2} (i + \tan [d + e x]) \right) \right] + \frac{b \operatorname{Log} \left[ 2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right]}{\sqrt{c}} + \right. \\
& \left. \sqrt{a - i b - c} \operatorname{Log} \left[ \left( b (i + \tan [d + e x]) + 2 \left( c + i a \tan [d + e x] - i \sqrt{a - i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right] / \right. \\
& \left. \left( (a - i b - c)^{3/2} (-i + \tan [d + e x]) \right) \right] \left. \operatorname{Tan}[d + e x] (b \operatorname{Csc}[d + e x]^2 - 2 \operatorname{Cot}[d + e x] \operatorname{Csc}[d + e x]^2 (c + b \tan [d + e x])) \right] / \right. \\
& \left. \left( 4 \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \sqrt{a + \cot [d + e x]^2 (c + b \tan [d + e x])} \right) - \frac{1}{2 \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right. \\
& \left. \operatorname{Tan}[d + e x] \sqrt{a + \cot [d + e x]^2 (c + b \tan [d + e x])} \right. \\
& \left( -\frac{b \operatorname{Csc}[d + e x] \operatorname{Sec}[d + e x]}{\sqrt{c}} + \frac{b \left( b \operatorname{Sec}[d + e x]^2 + \frac{\sqrt{c} (a \operatorname{Sec}[d + e x]^2 \operatorname{Tan}[d + e x] + \operatorname{Sec}[d + e x]^2 (b + a \tan [d + e x]))}{\sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right)}{\sqrt{c} \left( 2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right)} + \left( i (a + i b - c)^2 (i + \tan [d + e x]) \right. \right. \\
& \left. \left. \left( \left( i \left( 2 a \operatorname{Sec}[d + e x]^2 + i b \operatorname{Sec}[d + e x]^2 + \frac{\sqrt{a + i b - c} (a \operatorname{Sec}[d + e x]^2 \operatorname{Tan}[d + e x] + \operatorname{Sec}[d + e x]^2 (b + a \tan [d + e x]))}{\sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right) \right) \right) \right] / \right. \\
& \left. \left( (a + i b - c)^{3/2} (i + \tan [d + e x]) \right) - \left( i \operatorname{Sec}[d + e x]^2 \left( b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + \right. \right. \right. \\
& \left. \left. \left. 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right) / \left. \left( (a + i b - c)^{3/2} (i + \tan [d + e x])^2 \right) \right) \right] /
\end{aligned}$$



$$\begin{aligned}
& \left( b + 2i c + 2 a \tan[d + e x] + i b \tan[d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])} \right) + \left( (a - i b - c)^2 (-i + \tan[d + e x]) \right. \\
& \left. \left( \left( b \sec[d + e x]^2 + 2 \left( i a \sec[d + e x]^2 - \frac{i \sqrt{a - i b - c} (a \sec[d + e x]^2 \tan[d + e x] + \sec[d + e x]^2 (b + a \tan[d + e x]))}{2 \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])}} \right) \right) \right) / \right. \\
& \left. \left( (a - i b - c)^{3/2} (-i + \tan[d + e x]) \right) - \left( \sec[d + e x]^2 (b (i + \tan[d + e x]) + \right. \right. \\
& \left. \left. 2 (c + i a \tan[d + e x] - i \sqrt{a - i b - c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])}) \right) \right) / \left( (a - i b - c)^{3/2} (-i + \tan[d + e x])^2 \right) \right) / \\
& \left. \left( b (i + \tan[d + e x]) + 2 (c + i a \tan[d + e x] - i \sqrt{a - i b - c} \sqrt{c + \tan[d + e x] (b + a \tan[d + e x])}) \right) \right) \right) \right)
\end{aligned}$$

**Problem 9: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cot[d + e x] + c \cot[d + e x]^2} \tan[d + e x] dx$$

Optimal (type 3, 570 leaves, 18 steps):

$$\begin{aligned}
& \left( \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) \Bigg) / \\
& \quad \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} e \right) + \frac{\sqrt{a} \text{ArcTanh} \left[ \frac{2a + b \cot [d + ex]}{2\sqrt{a} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{e} - \\
& \left( \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) \Bigg) / \left( \sqrt{2 \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4}} e \right)
\end{aligned}$$

Result (type 3, 2361 leaves):

$$\begin{aligned}
& \left( \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \left( 2\sqrt{a} \text{Log} \left[ b + 2a \tan [d + ex] + 2\sqrt{a} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right] - \right. \right. \\
& \quad \left. \left. \sqrt{a + ib - c} \text{Log} \left[ 2i \left( b + 2ic + 2a \tan [d + ex] + ib \tan [d + ex] + 2\sqrt{a + ib - c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right) \right] \right) \right) / \\
& \quad \left( (a + ib - c)^{3/2} (i + \tan [d + ex]) \right) + \\
& \quad \left( \sqrt{a - ib - c} \text{Log} \left[ 2b (i + \tan [d + ex]) + 4 \left( c + ia \tan [d + ex] - i\sqrt{a - ib - c} \sqrt{c + \tan [d + ex] (b + a \tan [d + ex])} \right) \right] \right) / \\
& \quad \left( (a - ib - c)^{3/2} (-i + \tan [d + ex]) \right) \Bigg) \\
& \sqrt{\left( -\frac{a}{-1 + \cos [2 (d + ex)]} - \frac{c}{-1 + \cos [2 (d + ex)]} + \frac{a \cos [2 (d + ex)]}{-1 + \cos [2 (d + ex)]} - \frac{c \cos [2 (d + ex)]}{-1 + \cos [2 (d + ex)]} - \frac{b \sin [2 (d + ex)]}{-1 + \cos [2 (d + ex)]} \right) \tan [d + ex]^2} /
\end{aligned}$$

$$\begin{aligned}
& \left( 2 e^{\sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])} \right) \left( -\frac{1}{4 (c + \tan[d + ex] (b + a \tan[d + ex]))^{3/2}} \right. \\
& \sqrt{a + b \cot[d + ex] + c \cot[d + ex]^2} \left( 2 \sqrt{a} \log[b + 2 a \tan[d + ex] + 2 \sqrt{a} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])] - \right. \\
& \left. \sqrt{a + i b - c} \log \left[ \left( 2 i (b + 2 i c + 2 a \tan[d + ex] + i b \tan[d + ex] + 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right] \right) / \\
& \left( (a + i b - c)^{3/2} (i + \tan[d + ex]) \right) + \sqrt{a - i b - c} \log \left[ \left( 2 b (i + \tan[d + ex]) + 4 \right. \right. \\
& \left. \left. (c + i a \tan[d + ex] - i \sqrt{a - i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right] / \left( (a - i b - c)^{3/2} (-i + \tan[d + ex]) \right) \Big] \\
& \tan[d + ex] (a \sec[d + ex]^2 \tan[d + ex] + \sec[d + ex]^2 (b + a \tan[d + ex])) + \frac{1}{2 \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])} \\
& \sqrt{a + b \cot[d + ex] + c \cot[d + ex]^2} \left( 2 \sqrt{a} \log[b + 2 a \tan[d + ex] + 2 \sqrt{a} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])] - \right. \\
& \left. \sqrt{a + i b - c} \log \left[ \left( 2 i (b + 2 i c + 2 a \tan[d + ex] + i b \tan[d + ex] + 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right] \right) / \\
& \left( (a + i b - c)^{3/2} (i + \tan[d + ex]) \right) + \sqrt{a - i b - c} \log \left[ \left( 2 b (i + \tan[d + ex]) + 4 (c + i a \tan[d + ex] - \right. \right. \\
& \left. \left. i \sqrt{a - i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right] / \left( (a - i b - c)^{3/2} (-i + \tan[d + ex]) \right) \Big] \sec[d + ex]^2 + \\
& \left( (-b \csc[d + ex]^2 - 2 c \cot[d + ex] \csc[d + ex]^2) \left( 2 \sqrt{a} \log[b + 2 a \tan[d + ex] + 2 \sqrt{a} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])] - \right. \right. \\
& \left. \left. \sqrt{a + i b - c} \log \left[ \left( 2 i (b + 2 i c + 2 a \tan[d + ex] + i b \tan[d + ex] + 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right] \right) \right) / \\
& \left( (a + i b - c)^{3/2} (i + \tan[d + ex]) \right) + \sqrt{a - i b - c} \log \left[ \left( 2 b (i + \tan[d + ex]) + 4 (c + i a \tan[d + ex] - \right. \right. \\
& \left. \left. i \sqrt{a - i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right] / \left( (a - i b - c)^{3/2} (-i + \tan[d + ex]) \right) \Big] \tan[d + ex] \Big) / \\
& \left( 4 \sqrt{a + b \cot[d + ex] + c \cot[d + ex]^2} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex]) \right) + \frac{1}{2 \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])} \\
& \sqrt{a + b \cot[d + ex] + c \cot[d + ex]^2} \tan[d + ex] \\
& \left( \frac{2 \sqrt{a} \left( 2 a \sec[d + ex]^2 + \frac{\sqrt{a} (a \sec[d + ex]^2 \tan[d + ex] + \sec[d + ex]^2 (b + a \tan[d + ex]))}{\sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])} \right)}{b + 2 a \tan[d + ex] + 2 \sqrt{a} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])} \right) + \left( i (a + i b - c)^2 (i + \tan[d + ex]) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( 2i \left( 2a \operatorname{Sec}[d+ex]^2 + ib \operatorname{Sec}[d+ex]^2 + \frac{\sqrt{a+ib-c} (a \operatorname{Sec}[d+ex]^2 \operatorname{Tan}[d+ex] + \operatorname{Sec}[d+ex]^2 (b+a \operatorname{Tan}[d+ex]))}{\sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex])} \right) \right) \right) / \\
& \left( (a+ib-c)^{3/2} (i+\operatorname{Tan}[d+ex]) \right) - \left( 2i \operatorname{Sec}[d+ex]^2 (b+2ic+2a \operatorname{Tan}[d+ex] + ib \operatorname{Tan}[d+ex] + 2\sqrt{a+ib-c} \right. \\
& \left. \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \Big) / \left( (a+ib-c)^{3/2} (i+\operatorname{Tan}[d+ex])^2 \right) \Big) / \left( 2 (b+2ic+2a \operatorname{Tan}[d+ex] + \right. \\
& \left. ib \operatorname{Tan}[d+ex] + 2\sqrt{a+ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \Big) + \left( (a-ib-c)^2 (-i+\operatorname{Tan}[d+ex]) \right. \\
& \left. \left( \left( 2b \operatorname{Sec}[d+ex]^2 + 4 \left( ia \operatorname{Sec}[d+ex]^2 - \frac{i\sqrt{a-ib-c} (a \operatorname{Sec}[d+ex]^2 \operatorname{Tan}[d+ex] + \operatorname{Sec}[d+ex]^2 (b+a \operatorname{Tan}[d+ex]))}{2\sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex])} \right) \right) \right) / \right. \\
& \left. \left( (a-ib-c)^{3/2} (-i+\operatorname{Tan}[d+ex]) \right) - \left( \operatorname{Sec}[d+ex]^2 (2b (i+\operatorname{Tan}[d+ex]) + \right. \right. \\
& \left. \left. 4 (c+ia \operatorname{Tan}[d+ex] - i\sqrt{a-ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \right) \Big) / \left( (a-ib-c)^{3/2} (-i+\operatorname{Tan}[d+ex])^2 \right) \Big) \Big) / \\
& \left( 2b (i+\operatorname{Tan}[d+ex]) + 4 (c+ia \operatorname{Tan}[d+ex] - i\sqrt{a-ib-c} \sqrt{c+\operatorname{Tan}[d+ex]} (b+a \operatorname{Tan}[d+ex]) \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

**Problem 10: Humongous result has more than 200000 leaves.**

$$\int \sqrt{a+b \operatorname{Cot}[d+ex] + c \operatorname{Cot}[d+ex]^2} \operatorname{Tan}[d+ex]^3 dx$$

Optimal (type 3, 691 leaves, 21 steps):

$$\begin{aligned}
& - \left( \left( \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \right. \\
& \quad \left. \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \right. \\
& \quad \left. \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) - \frac{\sqrt{a} \text{ArcTanh} \left[ \frac{2a + b \cot [d + ex]}{2\sqrt{a} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{e} - \frac{(b^2 - 4ac) \text{ArcTanh} \left[ \frac{2a + b \cot [d + ex]}{2\sqrt{a} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{8a^{3/2} e} + \\
& \quad \left( \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \cot [d + ex] \right) \right] \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \right) / \\
& \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) + \frac{(2a + b \cot [d + ex]) \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2} \tan [d + ex]^2}{4ae}
\end{aligned}$$

Result (type ?, 465721 leaves): Display of huge result suppressed!

**Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot [d + ex]^7}{(a + b \cot [d + ex] + c \cot [d + ex]^2)^{3/2}} dx$$

Optimal (type 3, 1189 leaves, 20 steps):

$$\begin{aligned}
& - \frac{3b \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{2c^{5/2} e} + \frac{5b(7b^2 - 12ac) \text{ArcTanh} \left[ \frac{b + 2c \cot [d + ex]}{2\sqrt{c} \sqrt{a + b \cot [d + ex] + c \cot [d + ex]^2}} \right]}{16c^{9/2} e} + \\
& \quad \left( \sqrt{2a - 2c - \sqrt{a^2 + b^2 - 2ac + c^2}} \sqrt{a^2 - b^2 - 2ac + c^2} + (a - c) \sqrt{a^2 + b^2 - 2ac + c^2} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 - (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \left( 2a - 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \cot [d + ex] \right) \right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{2} \sqrt{2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2 + (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2} \right) / \\
& \left( \sqrt{2} (a^2 + b^2 - 2 a c + c^2)^{3/2} e \right) - \left( \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2 - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2}} \right. \\
& \left. \operatorname{ArcTanh} \left[ \left( b^2 - (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - b \left( 2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \operatorname{Cot}[d + e x] \right) \right] / \right. \\
& \left. \left( \sqrt{2} \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2 - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2} \right) \right) / \\
& \left( \sqrt{2} (a^2 + b^2 - 2 a c + c^2)^{3/2} e \right) - \frac{2 (2 a + b \operatorname{Cot}[d + e x])}{(b^2 - 4 a c) e \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}} + \\
& \frac{2 \operatorname{Cot}[d + e x]^2 (2 a + b \operatorname{Cot}[d + e x])}{(b^2 - 4 a c) e \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}} - \frac{2 \operatorname{Cot}[d + e x]^4 (2 a + b \operatorname{Cot}[d + e x])}{(b^2 - 4 a c) e \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}} + \\
& \frac{2 (a (b^2 - 2 (a - c) c) + b c (a + c) \operatorname{Cot}[d + e x])}{(b^2 + (a - c)^2) (b^2 - 4 a c) e \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}} - \\
& \frac{(7 b^2 - 16 a c) \operatorname{Cot}[d + e x]^2 \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}}{3 c^2 (b^2 - 4 a c) e} + \\
& \frac{2 b \operatorname{Cot}[d + e x]^3 \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}}{c (b^2 - 4 a c) e} + \\
& \frac{(3 b^2 - 8 a c - 2 b c \operatorname{Cot}[d + e x]) \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}}{c^2 (b^2 - 4 a c) e} - \\
& \frac{(105 b^4 - 460 a b^2 c + 256 a^2 c^2 - 2 b c (35 b^2 - 116 a c) \operatorname{Cot}[d + e x]) \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2}}{24 c^4 (b^2 - 4 a c) e}
\end{aligned}$$

Result (type 3, 5618 leaves):

$$\frac{1}{e} \sqrt{\frac{-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}$$

$$\left( (105 a^3 b^4 + 105 a b^6 - 460 a^4 b^2 c - 727 a^2 b^4 c - 57 b^6 c + 256 a^5 c^2 + 1364 a^3 b^2 c^2 + 407 a b^4 c^2 - 448 a^4 c^3 - 740 a^2 b^2 c^3 - \right.$$

$$\begin{aligned}
& \frac{25 b^4 c^3 + 96 a^3 c^4 + 44 a b^2 c^4 + 224 a^2 c^5 + 32 b^2 c^5 - 128 a c^6}{(24 (a - c) (a - i b - c) (a + i b - c) c^4 (-b^2 + 4 a c))} + \\
& \frac{11 b \operatorname{Cot}[d + e x] - \operatorname{Csc}[d + e x]^2}{12 c^3} - \frac{\operatorname{Csc}[d + e x]^2}{3 c^2} + (2 (2 a^3 b^4 + 2 a b^6 - 8 a^4 b^2 c - 12 a^2 b^4 c + 4 a^5 c^2 + 18 a^3 b^2 c^2 - 4 a^4 c^3 + a^4 b^3 \operatorname{Sin}[2 (d + e x)] + \\
& 2 a^2 b^5 \operatorname{Sin}[2 (d + e x)] + b^7 \operatorname{Sin}[2 (d + e x)] - 3 a^5 b c \operatorname{Sin}[2 (d + e x)] - 10 a^3 b^3 c \operatorname{Sin}[2 (d + e x)] - \\
& 7 a b^5 c \operatorname{Sin}[2 (d + e x)] + 10 a^4 b c^2 \operatorname{Sin}[2 (d + e x)] + 14 a^2 b^3 c^2 \operatorname{Sin}[2 (d + e x)] - 7 a^3 b c^3 \operatorname{Sin}[2 (d + e x)])) / \\
& ((a - c) (a - i b - c) (a + i b - c) c^3 (-b^2 + 4 a c) (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])) + \\
& \left( \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2} \left( -b (i a + b - i c) (-i a + b + i c) (35 b^2 - 12 c (5 a + 2 c)) \operatorname{Log}[\operatorname{Tan}[d + e x]] + \right. \right. \\
& \frac{8 (a + i b - c) c^{9/2} \operatorname{Log}\left[\frac{i b + 2 c + (2 i a + b) \operatorname{Tan}[d + e x] - 2 i \sqrt{a - i b - c} \sqrt{c + b \operatorname{Tan}[d + e x] + a \operatorname{Tan}[d + e x]^2}}{8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \operatorname{Tan}[d + e x])}\right]}{\sqrt{a - i b - c}} + \\
& \frac{8 c^{9/2} (-a + i b + c) \operatorname{Log}\left[\frac{i (b + 2 i c + 2 a \operatorname{Tan}[d + e x] + i b \operatorname{Tan}[d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \operatorname{Tan}[d + e x] (b + a \operatorname{Tan}[d + e x])})}{8 (a - i b - c) \sqrt{a + i b - c} c^4 (i + \operatorname{Tan}[d + e x])}\right]}{\sqrt{a + i b - c}} + \\
& \left. \left. b (i a + b - i c) (-i a + b + i c) (35 b^2 - 12 c (5 a + 2 c)) \operatorname{Log}\left[2 c + b \operatorname{Tan}[d + e x] + 2 \sqrt{c} \sqrt{c + \operatorname{Tan}[d + e x] (b + a \operatorname{Tan}[d + e x])}\right] \right) \right) \\
& \left( - \frac{2 b \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{(a - i b - c) (a + i b - c) (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} + \right. \\
& \frac{35 a^2 b^3 \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{8 (a - i b - c) (a + i b - c) c^4 (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} + \\
& \left. \frac{35 b^5 \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{8 (a - i b - c) (a + i b - c) c^4 (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{15 a^3 b \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{2(a-i b-c)(a+i b-c) c^3(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& - \frac{65 a b^3 \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{4(a-i b-c)(a+i b-c) c^3(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& + \frac{12 a^2 b \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{(a-i b-c)(a+i b-c) c^2(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& + \frac{11 b^3 \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{8(a-i b-c)(a+i b-c) c^2(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& - \frac{3 a b \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{2(a-i b-c)(a+i b-c) c(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& + \frac{b \cos[2(d+e x)] \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{(a-i b-c)(a+i b-c)(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& - \frac{a \sin[2(d+e x)] \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{(a-i b-c)(a+i b-c)(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \\
& \left. \frac{c \sin[2(d+e x)] \sqrt{-\frac{a}{-1+\cos[2(d+e x)]} - \frac{c}{-1+\cos[2(d+e x)]} + \frac{a \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{c \cos[2(d+e x)]}{-1+\cos[2(d+e x)]} - \frac{b \sin[2(d+e x)]}{-1+\cos[2(d+e x)]}}{(a-i b-c)(a+i b-c)(-a-c+a \cos[2(d+e x)]-c \cos[2(d+e x)]-b \sin[2(d+e x)])} \right) \tan[d+e x] \Big/ \\
& \left( 16 c^{9/2} (a^2+b^2-2 a c+c^2) e \sqrt{c+\tan[d+e x]} (b+a \tan[d+e x]) \right)
\end{aligned}$$



$$\left( -\frac{1}{32 c^{9/2} (a^2 + b^2 - 2 a c + c^2) (c + \tan [d + e x] (b + a \tan [d + e x]))^{3/2}} \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \right. \\
\left. \frac{8 (a + i b - c) c^{9/2} \operatorname{Log}\left[\frac{i b + 2 c + (2 i a + b) \tan [d + e x] - 2 i \sqrt{a - i b - c} \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2}}{8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \tan [d + e x])}\right]}{(35 b^2 - 12 c (5 a + 2 c)) \operatorname{Log}[\tan [d + e x]] + \frac{\sqrt{a - i b - c}}{8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \tan [d + e x])}} + \right. \\
\left. \frac{8 c^{9/2} (-a + i b + c) \operatorname{Log}\left[\frac{i (b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])})}{8 (a - i b - c) \sqrt{a + i b - c} c^4 (i + \tan [d + e x])}\right]}{\sqrt{a + i b - c}} + b (i a + b - i c) \right. \\
\left. \frac{(-i a + b + i c) (35 b^2 - 12 c (5 a + 2 c)) \operatorname{Log}\left[2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}\right]}{(a \sec [d + e x]^2 \tan [d + e x] + \sec [d + e x]^2 (b + a \tan [d + e x])) + \frac{1}{16 c^{9/2} (a^2 + b^2 - 2 a c + c^2) \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right] \tan [d + e x]} \\
\left. \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \right) \\
\left( -b (i a + b - i c) (-i a + b + i c) (35 b^2 - 12 c (5 a + 2 c)) \operatorname{Log}[\tan [d + e x]] + \right. \\
\left. \frac{8 (a + i b - c) c^{9/2} \operatorname{Log}\left[\frac{i b + 2 c + (2 i a + b) \tan [d + e x] - 2 i \sqrt{a - i b - c} \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2}}{8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \tan [d + e x])}\right]}{\sqrt{a - i b - c}} + \right. \\
\left. \frac{8 c^{9/2} (-a + i b + c) \operatorname{Log}\left[\frac{i (b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])})}{8 (a - i b - c) \sqrt{a + i b - c} c^4 (i + \tan [d + e x])}\right]}{\sqrt{a + i b - c}} + b (i a + b - i c) (-i a + b + i c) \right. \\
\left. \frac{(35 b^2 - 12 c (5 a + 2 c)) \operatorname{Log}\left[2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}\right]}{(a \sec [d + e x]^2 \tan [d + e x] + \sec [d + e x]^2 (b + a \tan [d + e x])) + \frac{1}{16 c^{9/2} (a^2 + b^2 - 2 a c + c^2) \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right] \sec [d + e x]^2 + \right. \\
\left. \frac{1}{16 c^{9/2} (a^2 + b^2 - 2 a c + c^2) \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right)$$

$$\begin{aligned}
& \left( (-b \operatorname{Csc}[d+ex]^2 - 2c \operatorname{Cot}[d+ex] \operatorname{Csc}[d+ex]^2) \left( -b(i a + b - i c) (-i a + b + i c) (35b^2 - 12c(5a + 2c)) \operatorname{Log}[\operatorname{Tan}[d+ex]] + \right. \right. \\
& \frac{8(a + i b - c) c^{9/2} \operatorname{Log}\left[\frac{i b + 2c + (2i a + b) \operatorname{Tan}[d+ex] - 2i \sqrt{a - i b - c} \sqrt{c + b \operatorname{Tan}[d+ex] + a \operatorname{Tan}[d+ex]^2}}{8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \operatorname{Tan}[d+ex])}\right]}{\sqrt{a - i b - c}} + \\
& \frac{8c^{9/2} (-a + i b + c) \operatorname{Log}\left[\frac{i(b + 2i c + 2a \operatorname{Tan}[d+ex] + i b \operatorname{Tan}[d+ex] + 2\sqrt{a + i b - c} \sqrt{c + \operatorname{Tan}[d+ex]} (b + a \operatorname{Tan}[d+ex]))}{8(a - i b - c) \sqrt{a + i b - c} c^4 (i + \operatorname{Tan}[d+ex])}\right]}{\sqrt{a + i b - c}} + b(i a + b - i c) (-i a + b + i c) \\
& \left. \left. (35b^2 - 12c(5a + 2c)) \operatorname{Log}\left[2c + b \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{c + \operatorname{Tan}[d+ex]} (b + a \operatorname{Tan}[d+ex])\right]\right) \operatorname{Tan}[d+ex] \right) / \\
& \frac{\left(32c^{9/2} (a^2 + b^2 - 2ac + c^2) \sqrt{a + b \operatorname{Cot}[d+ex] + c \operatorname{Cot}[d+ex]^2} \sqrt{c + \operatorname{Tan}[d+ex]} (b + a \operatorname{Tan}[d+ex])\right) + 1}{16c^{9/2} (a^2 + b^2 - 2ac + c^2) \sqrt{c + \operatorname{Tan}[d+ex]} (b + a \operatorname{Tan}[d+ex])} \sqrt{a + b \operatorname{Cot}[d+ex] + c \operatorname{Cot}[d+ex]^2} \operatorname{Tan}[d+ex] \\
& \left( -b(i a + b - i c) (-i a + b + i c) (35b^2 - 12c(5a + 2c)) \operatorname{Csc}[d+ex] \operatorname{Sec}[d+ex] + \left( b(i a + b - i c) (-i a + b + i c) \right. \right. \\
& \left. \left. (35b^2 - 12c(5a + 2c)) \left( b \operatorname{Sec}[d+ex]^2 + \frac{\sqrt{c} (a \operatorname{Sec}[d+ex]^2 \operatorname{Tan}[d+ex] + \operatorname{Sec}[d+ex]^2 (b + a \operatorname{Tan}[d+ex]))}{\sqrt{c + \operatorname{Tan}[d+ex]} (b + a \operatorname{Tan}[d+ex])} \right) \right) / \\
& \left( 2c + b \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{c + \operatorname{Tan}[d+ex]} (b + a \operatorname{Tan}[d+ex]) \right) + \\
& \left( 64(a + i b - c)^2 c^{17/2} (-i + \operatorname{Tan}[d+ex]) \left( \frac{(2i a + b) \operatorname{Sec}[d+ex]^2 - \frac{i \sqrt{a - i b - c} (b \operatorname{Sec}[d+ex]^2 + 2a \operatorname{Sec}[d+ex]^2 \operatorname{Tan}[d+ex])}{\sqrt{c + b \operatorname{Tan}[d+ex] + a \operatorname{Tan}[d+ex]^2}}}{8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \operatorname{Tan}[d+ex])} - \right. \right. \\
& \left. \left. \left( \operatorname{Sec}[d+ex]^2 \left( i b + 2c + (2i a + b) \operatorname{Tan}[d+ex] - 2i \sqrt{a - i b - c} \sqrt{c + b \operatorname{Tan}[d+ex] + a \operatorname{Tan}[d+ex]^2} \right) \right) \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( \left( 8 \sqrt{a - i b - c} (a + i b - c) c^4 (-i + \tan[d + e x])^2 \right) \right) / \left( i b + 2 c + (2 i a + b) \tan[d + e x] - \right. \right. \\
& \left. \left. 2 i \sqrt{a - i b - c} \sqrt{c + b \tan[d + e x] + a \tan[d + e x]^2} \right) - \left( 64 i (a - i b - c) c^{17/2} (-a + i b + c) (i + \tan[d + e x]) \right. \right. \\
& \left. \left. \left( \left( i \left( 2 a \sec[d + e x]^2 + i b \sec[d + e x]^2 + \frac{\sqrt{a + i b - c} (a \sec[d + e x]^2 \tan[d + e x] + \sec[d + e x]^2 (b + a \tan[d + e x]))}{\sqrt{c + \tan[d + e x]} (b + a \tan[d + e x])} \right) \right) \right) / \right. \right. \\
& \left. \left( 8 (a - i b - c) \sqrt{a + i b - c} c^4 (i + \tan[d + e x]) \right) - \left( i \sec[d + e x]^2 (b + 2 i c + 2 a \tan[d + e x] + i b \tan[d + e x] + \right. \right. \\
& \left. \left. 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + e x]} (b + a \tan[d + e x]) \right) \right) / \left( 8 (a - i b - c) \sqrt{a + i b - c} c^4 (i + \tan[d + e x])^2 \right) \right) \left. \right) / \\
& \left. \left( b + 2 i c + 2 a \tan[d + e x] + i b \tan[d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan[d + e x]} (b + a \tan[d + e x]) \right) \right) \left. \right) \left. \right)
\end{aligned}$$

**Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[d + e x]^5}{(a + b \cot[d + e x] + c \cot[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 865 leaves, 14 steps):

$$\begin{aligned}
& \frac{3 b \operatorname{ArcTanh}\left[\frac{b+2 c \cot [d+e x]}{2 \sqrt{c} \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}}\right]}{2 c^{5/2} e} - \left( \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \right. \\
& \left. \operatorname{ArcTanh}\left[\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}\right) \cot [d+e x]\right) / \right. \right. \\
& \left. \left. \left(\sqrt{2} \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}\right)\right] \right) / \\
& \left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{3/2} e\right) + \left(\sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \right. \\
& \left. \operatorname{ArcTanh}\left[\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}\right) \cot [d+e x]\right) / \right. \right. \\
& \left. \left. \left(\sqrt{2} \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}\right)\right] \right) / \\
& \left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{3/2} e\right) + \frac{2(2 a+b \cot [d+e x])}{\left(b^2-4 a c\right) e \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}} - \\
& \frac{2 \cot [d+e x]^2(2 a+b \cot [d+e x])}{\left(b^2-4 a c\right) e \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}} - \\
& \frac{2\left(a\left(b^2-2(a-c) c\right)+b c(a+c) \cot [d+e x]\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4 a c\right) e \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}} - \\
& \frac{\left(3 b^2-8 a c-2 b c \cot [d+e x]\right) \sqrt{a+b \cot [d+e x]+c \cot [d+e x]^2}}{c^2\left(b^2-4 a c\right) e}
\end{aligned}$$

Result (type 3, 4537 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{\frac{-a-c+a \cos [2(d+e x)]-c \cos [2(d+e x)]-b \sin [2(d+e x)]}{-1+\cos [2(d+e x)]}} \\
& \left(-\frac{-3 a^3 b^2-3 a b^4+8 a^4 c+15 a^2 b^2 c+b^4 c-16 a^3 c^2-7 a b^2 c^2+12 a^2 c^3+b^2 c^3-4 a c^4}{(a-c)(a-i b-c)(a+i b-c) c^2(-b^2+4 a c)} - \right. \\
& \left. \left(2\left(-2 a^3 b^2-2 a b^4+4 a^4 c+8 a^2 b^2 c-4 a^3 c^2-a^4 b \sin [2(d+e x)]-2 a^2 b^3 \sin [2(d+e x)]- \right. \right. \right. \\
& \left. \left. \left. b^5 \sin [2(d+e x)]+6 a^3 b c \sin [2(d+e x)]+5 a b^3 c \sin [2(d+e x)]-5 a^2 b c^2 \sin [2(d+e x)]\right)\right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left( (a - c) (a - i b - c) (a + i b - c) c (-b^2 + 4 a c) (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)]) \right) - \\
& \left( \sqrt{a + b \operatorname{Cot}[d + e x] + c \operatorname{Cot}[d + e x]^2} \left( 3 b (i a + b - i c) (-i a + b + i c) \operatorname{Log}[\operatorname{Tan}[d + e x]] + \right. \right. \\
& \frac{(a + i b - c) c^{5/2} \operatorname{Log}\left[\frac{i b + 2 c + (2 i a + b) \operatorname{Tan}[d + e x] - 2 i \sqrt{a - i b - c} \sqrt{c + b \operatorname{Tan}[d + e x] + a \operatorname{Tan}[d + e x]^2}}{\sqrt{a - i b - c} (a + i b - c) c^2 (-i + \operatorname{Tan}[d + e x])}\right]}{\sqrt{a - i b - c}} + \\
& \left. \frac{c^{5/2} (-a + i b + c) \operatorname{Log}\left[\frac{i (b + 2 i c + 2 a \operatorname{Tan}[d + e x] + i b \operatorname{Tan}[d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \operatorname{Tan}[d + e x] (b + a \operatorname{Tan}[d + e x])})}{(a - i b - c) \sqrt{a + i b - c} c^2 (i + \operatorname{Tan}[d + e x])}\right]}{\sqrt{a + i b - c}} \right) - \\
& \left. 3 b (i a + b - i c) (-i a + b + i c) \operatorname{Log}\left[2 c + b \operatorname{Tan}[d + e x] + 2 \sqrt{c} \sqrt{c + \operatorname{Tan}[d + e x] (b + a \operatorname{Tan}[d + e x])}\right] \right) \\
& \left( \frac{2 b \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{(a - i b - c) (a + i b - c) (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} + \right. \\
& \frac{3 a^2 b \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{(a - i b - c) (a + i b - c) c^2 (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} + \\
& \frac{3 b^3 \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{(a - i b - c) (a + i b - c) c^2 (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} - \\
& \left. \frac{6 a b \sqrt{-\frac{a}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c}{-1 + \operatorname{Cos}[2 (d + e x)]} + \frac{a \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{c \operatorname{Cos}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]} - \frac{b \operatorname{Sin}[2 (d + e x)]}{-1 + \operatorname{Cos}[2 (d + e x)]}}{(a - i b - c) (a + i b - c) c (-a - c + a \operatorname{Cos}[2 (d + e x)] - c \operatorname{Cos}[2 (d + e x)] - b \operatorname{Sin}[2 (d + e x)])} + \right)
\end{aligned}$$

$$\frac{b \cos[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{(a-ib-c)(a+ib-c)(-a-c+a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} -$$

$$\frac{a \sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{(a-ib-c)(a+ib-c)(-a-c+a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} +$$

$$\frac{c \sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}{(a-ib-c)(a+ib-c)(-a-c+a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} \left. \begin{array}{l} \text{Tan}[d+ex] \\ \text{---} \\ \text{---} \end{array} \right)$$

$$\left( 2 c^{5/2} (a^2 + b^2 - 2 a c + c^2) e \sqrt{c + \text{Tan}[d+ex]} (b + a \text{Tan}[d+ex]) \right)$$

$$\left( \frac{1}{4 c^{5/2} (a^2 + b^2 - 2 a c + c^2) (c + \text{Tan}[d+ex]) (b + a \text{Tan}[d+ex])} \sqrt{a + b \text{Cot}[d+ex] + c \text{Cot}[d+ex]^2} \right)^{3/2}$$

$$\left( 3 b (i a + b - i c) (-i a + b + i c) \text{Log}[\text{Tan}[d+ex]] + \frac{(a + i b - c) c^{5/2} \text{Log}\left[\frac{i b + 2 c + (2 i a + b) \text{Tan}[d+ex] - 2 i \sqrt{a - i b - c} \sqrt{c + b \text{Tan}[d+ex] + a \text{Tan}[d+ex]^2}}{\sqrt{a - i b - c} (a + i b - c) c^2 (-i + \text{Tan}[d+ex])}\right]}{\sqrt{a - i b - c}} \right) +$$

$$\frac{c^{5/2} (-a + i b + c) \text{Log}\left[\frac{i (b + 2 i c + 2 a \text{Tan}[d+ex] + i b \text{Tan}[d+ex] + 2 \sqrt{a + i b - c} \sqrt{c + \text{Tan}[d+ex]} (b + a \text{Tan}[d+ex]))}{(a - i b - c) \sqrt{a + i b - c} c^2 (i + \text{Tan}[d+ex])}\right]}{\sqrt{a + i b - c}} -$$

$$3 b (i a + b - i c) (-i a + b + i c) \text{Log}\left[2 c + b \text{Tan}[d+ex] + 2 \sqrt{c} \sqrt{c + \text{Tan}[d+ex]} (b + a \text{Tan}[d+ex])\right]$$

$$\text{Tan}[d+ex] (a \text{Sec}[d+ex]^2 \text{Tan}[d+ex] + \text{Sec}[d+ex]^2 (b + a \text{Tan}[d+ex])) -$$

$$\begin{aligned}
& \frac{1}{2 c^{5/2} (a^2 + b^2 - 2 a c + c^2) \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x])} \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \\
& \left( 3 b (i a + b - i c) (-i a + b + i c) \log [\tan [d + e x]] + \frac{(a + i b - c) c^{5/2} \log \left[ \frac{i b + 2 c + (2 i a + b) \tan [d + e x] - 2 i \sqrt{a - i b - c} \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2}}{\sqrt{a - i b - c} (a + i b - c) c^2 (-i + \tan [d + e x])} \right]}{\sqrt{a - i b - c}} + \right. \\
& \left. \frac{c^{5/2} (-a + i b + c) \log \left[ \frac{i (b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x]))}{(a - i b - c) \sqrt{a + i b - c} c^2 (i + \tan [d + e x])} \right]}{\sqrt{a + i b - c}} - 3 b (i a + b - i c) (-i a + b + i c) \log \left[ \right. \right. \\
& \left. \left. 2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x]) \right] \right) \sec [d + e x]^2 - \left( (-b \csc [d + e x]^2 - 2 c \cot [d + e x] \csc [d + e x]^2) \right. \\
& \left. \left( 3 b (i a + b - i c) (-i a + b + i c) \log [\tan [d + e x]] + \frac{(a + i b - c) c^{5/2} \log \left[ \frac{i b + 2 c + (2 i a + b) \tan [d + e x] - 2 i \sqrt{a - i b - c} \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2}}{\sqrt{a - i b - c} (a + i b - c) c^2 (-i + \tan [d + e x])} \right]}{\sqrt{a - i b - c}} + \right. \right. \\
& \left. \left. \frac{c^{5/2} (-a + i b + c) \log \left[ \frac{i (b + 2 i c + 2 a \tan [d + e x] + i b \tan [d + e x] + 2 \sqrt{a + i b - c} \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x]))}{(a - i b - c) \sqrt{a + i b - c} c^2 (i + \tan [d + e x])} \right]}{\sqrt{a + i b - c}} - \right. \right. \\
& \left. \left. 3 b (i a + b - i c) (-i a + b + i c) \log \left[ 2 c + b \tan [d + e x] + 2 \sqrt{c} \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x]) \right] \right) \tan [d + e x] \right) / \\
& \left( 4 c^{5/2} (a^2 + b^2 - 2 a c + c^2) \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x]) \right) - \\
& \frac{1}{2 c^{5/2} (a^2 + b^2 - 2 a c + c^2) \sqrt{c + \tan [d + e x]} (b + a \tan [d + e x])} \\
& \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \tan [d + e x] \left( 3 b (i a + b - i c) (-i a + b + i c) \csc [d + e x] \sec [d + e x] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 3b (\mathbf{i}a + b - \mathbf{i}c) (-\mathbf{i}a + b + \mathbf{i}c) \left( b \operatorname{Sec}[d + ex]^2 + \frac{\sqrt{c} (a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \operatorname{Sec}[d + ex]^2 (b + a \operatorname{Tan}[d + ex]))}{\sqrt{c + \operatorname{Tan}[d + ex] (b + a \operatorname{Tan}[d + ex])}} \right) \right) / \\
& \left( 2c + b \operatorname{Tan}[d + ex] + 2\sqrt{c} \sqrt{c + \operatorname{Tan}[d + ex] (b + a \operatorname{Tan}[d + ex])} \right) + \left( (a + \mathbf{i}b - c)^2 c^{9/2} (-\mathbf{i} + \operatorname{Tan}[d + ex]) \right. \\
& \left. \frac{\left( (2\mathbf{i}a + b) \operatorname{Sec}[d + ex]^2 - \frac{\mathbf{i}\sqrt{a - \mathbf{i}b - c} (b \operatorname{Sec}[d + ex]^2 + 2a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex])}{\sqrt{c + b \operatorname{Tan}[d + ex] + a \operatorname{Tan}[d + ex]^2}} \right)}{\sqrt{a - \mathbf{i}b - c} (a + \mathbf{i}b - c) c^2 (-\mathbf{i} + \operatorname{Tan}[d + ex])} - \left( \operatorname{Sec}[d + ex]^2 (\mathbf{i}b + 2c + (2\mathbf{i}a + b) \operatorname{Tan}[d + ex]) - \right. \right. \\
& \left. \left. 2\mathbf{i}\sqrt{a - \mathbf{i}b - c} \sqrt{c + b \operatorname{Tan}[d + ex] + a \operatorname{Tan}[d + ex]^2} \right) \right) / \left( \sqrt{a - \mathbf{i}b - c} (a + \mathbf{i}b - c) c^2 (-\mathbf{i} + \operatorname{Tan}[d + ex])^2 \right) \left. \right) / (\mathbf{i}b + 2c + \\
& (2\mathbf{i}a + b) \operatorname{Tan}[d + ex] - 2\mathbf{i}\sqrt{a - \mathbf{i}b - c} \sqrt{c + b \operatorname{Tan}[d + ex] + a \operatorname{Tan}[d + ex]^2}) - \left( \mathbf{i}(a - \mathbf{i}b - c) c^{9/2} (-a + \mathbf{i}b + c) (\mathbf{i} + \operatorname{Tan}[d + ex]) \right) \\
& \left( \left( \mathbf{i} \left( 2a \operatorname{Sec}[d + ex]^2 + \mathbf{i}b \operatorname{Sec}[d + ex]^2 + \frac{\sqrt{a + \mathbf{i}b - c} (a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \operatorname{Sec}[d + ex]^2 (b + a \operatorname{Tan}[d + ex]))}{\sqrt{c + \operatorname{Tan}[d + ex] (b + a \operatorname{Tan}[d + ex])}} \right) \right) / \right. \\
& \left( (a - \mathbf{i}b - c) \sqrt{a + \mathbf{i}b - c} c^2 (\mathbf{i} + \operatorname{Tan}[d + ex]) \right) - \left( \mathbf{i} \operatorname{Sec}[d + ex]^2 (b + 2\mathbf{i}c + 2a \operatorname{Tan}[d + ex] + \mathbf{i}b \operatorname{Tan}[d + ex] + \right. \\
& \left. 2\sqrt{a + \mathbf{i}b - c} \sqrt{c + \operatorname{Tan}[d + ex] (b + a \operatorname{Tan}[d + ex])} \right) \left. \right) / \left( (a - \mathbf{i}b - c) \sqrt{a + \mathbf{i}b - c} c^2 (\mathbf{i} + \operatorname{Tan}[d + ex])^2 \right) \left. \right) / \\
& \left( b + 2\mathbf{i}c + 2a \operatorname{Tan}[d + ex] + \mathbf{i}b \operatorname{Tan}[d + ex] + 2\sqrt{a + \mathbf{i}b - c} \sqrt{c + \operatorname{Tan}[d + ex] (b + a \operatorname{Tan}[d + ex])} \right) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

**Problem 13:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cot}[d + ex]^3}{(a + b \operatorname{Cot}[d + ex] + c \operatorname{Cot}[d + ex]^2)^{3/2}} dx$$



Optimal (type 3, 686 leaves, 10 steps):

$$\left( \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\ \left. \operatorname{ArcTanh} \left[ \left( b^2-(a-c) \left( a-c+\sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c-\sqrt{a^2+b^2-2ac+c^2} \right) \cot [d+ex] \right) / \right. \right. \\ \left. \left. \left( \sqrt{2} \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \cot [d+ex]+c \cot [d+ex]^2} \right) \right] \right) / \\ \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{3/2} e \right) - \left( \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\ \left. \operatorname{ArcTanh} \left[ \left( b^2-(a-c) \left( a-c-\sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c+\sqrt{a^2+b^2-2ac+c^2} \right) \cot [d+ex] \right) / \right. \right. \\ \left. \left. \left( \sqrt{2} \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \cot [d+ex]+c \cot [d+ex]^2} \right) \right] \right) / \\ \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{3/2} e \right) - \frac{2(2a+b \cot [d+ex])}{(b^2-4ac) e \sqrt{a+b \cot [d+ex]+c \cot [d+ex]^2}} + \\ \frac{2(a(b^2-2(a-c)c)+bc(a+c) \cot [d+ex])}{(b^2+(a-c)^2)(b^2-4ac) e \sqrt{a+b \cot [d+ex]+c \cot [d+ex]^2}}$$

Result (type 3, 3282 leaves):

$$\frac{1}{e} \sqrt{\frac{-a-c+a \cos [2(d+ex)]-c \cos [2(d+ex)]-b \sin [2(d+ex)]}{-1+\cos [2(d+ex)]}} \\ \left( \frac{2a(2a^2+b^2-2ac)}{(a-c)(a+ib-c)(-ab^2+ib^3+4a^2c-4iabcb^2c-4ac^2)} + ((\cos [2(d+ex)]-i \sin [2(d+ex)]) \right. \\ \left. (ia^3b+2ia^2bc+ib^3c-3iabcb^2+8a^3c \cos [2(d+ex)]+4ab^2c \cos [2(d+ex)]-8a^2c^2 \cos [2(d+ex)]-ia^3b \cos [4(d+ex)]- \right. \\ \left. 2ia^2bc \cos [4(d+ex)]-ib^3c \cos [4(d+ex)]+3iabcb^2 \cos [4(d+ex)]+8ia^3c \sin [2(d+ex)]+4ia^2bc \sin [2(d+ex)]- \right. \\ \left. 8ia^2c^2 \sin [2(d+ex)]+a^3b \sin [4(d+ex)]+2a^2bc \sin [4(d+ex)]+b^3c \sin [4(d+ex)]-3iabcb^2 \sin [4(d+ex)])) / \right. \\ \left. ((a-c)(a-ib-c)(a+ib-c)(-b^2+4ac)(-a-c+a \cos [2(d+ex)]-c \cos [2(d+ex)]-b \sin [2(d+ex)])) \right) +$$



$$\begin{aligned}
& \left. \frac{\text{Log} \left[ \frac{4c+2b(-i+\text{Tan}[d+ex]) - 4i \left( a \text{Tan}[d+ex] + \sqrt{a+ib-c} \sqrt{c+\text{Tan}[d+ex]} (b+a \text{Tan}[d+ex]) \right)}{(a-ib-c) \sqrt{a+ib-c} (i+\text{Tan}[d+ex])}}{(a+ib-c)^{3/2}} \right]}{\text{Tan}[d+ex]} \right) \\
& \left. \left( b \text{Sec}[d+ex]^2 + 2a \text{Sec}[d+ex]^2 \text{Tan}[d+ex] \right) \right/ \left( 4(c+b \text{Tan}[d+ex] + a \text{Tan}[d+ex]^2)^{3/2} \right) + \\
& \left( \sqrt{a+b \text{Cot}[d+ex] + c \text{Cot}[d+ex]^2} \left( \frac{\text{Log} \left[ \frac{-4c-4ia \text{Tan}[d+ex] - 2b(i+\text{Tan}[d+ex]) + 4i \sqrt{a-ib-c} \sqrt{c+\text{Tan}[d+ex]} (b+a \text{Tan}[d+ex])}{\sqrt{a-ib-c} (a+ib-c) (-i+\text{Tan}[d+ex])}}{(a-ib-c)^{3/2}} \right]}{-} \right. \right. \\
& \left. \left. \frac{\text{Log} \left[ \frac{4c+2b(-i+\text{Tan}[d+ex]) - 4i \left( a \text{Tan}[d+ex] + \sqrt{a+ib-c} \sqrt{c+\text{Tan}[d+ex]} (b+a \text{Tan}[d+ex]) \right)}{(a-ib-c) \sqrt{a+ib-c} (i+\text{Tan}[d+ex])}}{(a+ib-c)^{3/2}} \right]}{\text{Sec}[d+ex]^2} \right) \right/ \left( 2 \sqrt{c+b \text{Tan}[d+ex] + a \text{Tan}[d+ex]^2} \right) + \\
& \left( -b \text{Csc}[d+ex]^2 - 2c \text{Cot}[d+ex] \text{Csc}[d+ex]^2 \right) \left( \frac{\text{Log} \left[ \frac{-4c-4ia \text{Tan}[d+ex] - 2b(i+\text{Tan}[d+ex]) + 4i \sqrt{a-ib-c} \sqrt{c+\text{Tan}[d+ex]} (b+a \text{Tan}[d+ex])}{\sqrt{a-ib-c} (a+ib-c) (-i+\text{Tan}[d+ex])}}{(a-ib-c)^{3/2}} \right]}{-} \right. \\
& \left. \frac{\text{Log} \left[ \frac{4c+2b(-i+\text{Tan}[d+ex]) - 4i \left( a \text{Tan}[d+ex] + \sqrt{a+ib-c} \sqrt{c+\text{Tan}[d+ex]} (b+a \text{Tan}[d+ex]) \right)}{(a-ib-c) \sqrt{a+ib-c} (i+\text{Tan}[d+ex])}}{(a+ib-c)^{3/2}} \right]}{\text{Tan}[d+ex]} \right) \right/ \\
& \left( 4 \sqrt{a+b \text{Cot}[d+ex] + c \text{Cot}[d+ex]^2} \sqrt{c+b \text{Tan}[d+ex] + a \text{Tan}[d+ex]^2} \right) + \frac{1}{2 \sqrt{c+b \text{Tan}[d+ex] + a \text{Tan}[d+ex]^2}} \\
& \sqrt{a+b \text{Cot}[d+ex] + c \text{Cot}[d+ex]^2} \text{Tan}[d+ex] \left( \left( (a+ib-c) (-i+\text{Tan}[d+ex]) \right. \right. \\
& \left. \left. \left( \left( -4ia \text{Sec}[d+ex]^2 - 2b \text{Sec}[d+ex]^2 + \frac{2i \sqrt{a-ib-c} (a \text{Sec}[d+ex]^2 \text{Tan}[d+ex] + \text{Sec}[d+ex]^2 (b+a \text{Tan}[d+ex]))}{\sqrt{c+\text{Tan}[d+ex]} (b+a \text{Tan}[d+ex])} \right) \right) \right) \right) \right/
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{a - i b - c} (a + i b - c) (-i + \tan[d + ex]) \right) - \left( \sec[d + ex]^2 \left( -4c - 4i a \tan[d + ex] - 2b (i + \tan[d + ex]) + 4i \sqrt{a - i b - c} \right. \right. \\
& \quad \left. \left. \sqrt{c + \tan[d + ex] (b + a \tan[d + ex])} \right) \right) / \left( \sqrt{a - i b - c} (a + i b - c) (-i + \tan[d + ex])^2 \right) \Bigg) / \left( (a - i b - c) (-4c - 4i a \right. \\
& \quad \left. \tan[d + ex] - 2b (i + \tan[d + ex]) + 4i \sqrt{a - i b - c} \sqrt{c + \tan[d + ex] (b + a \tan[d + ex])} \right) \Bigg) - \left( (a - i b - c) (i + \tan[d + ex]) \right. \\
& \quad \left( \left( 2b \sec[d + ex]^2 - 4i \left( a \sec[d + ex]^2 + \frac{\sqrt{a + i b - c} (a \sec[d + ex]^2 \tan[d + ex] + \sec[d + ex]^2 (b + a \tan[d + ex]))}{2 \sqrt{c + \tan[d + ex] (b + a \tan[d + ex])}} \right) \right) \right) / \\
& \quad \left( (a - i b - c) \sqrt{a + i b - c} (i + \tan[d + ex]) \right) - \left( \sec[d + ex]^2 \left( 4c + 2b (-i + \tan[d + ex]) - 4i (a \tan[d + ex] + \right. \right. \\
& \quad \left. \left. \sqrt{a + i b - c} \sqrt{c + \tan[d + ex] (b + a \tan[d + ex])} \right) \right) \Bigg) / \left( (a - i b - c) \sqrt{a + i b - c} (i + \tan[d + ex])^2 \right) \Bigg) / \\
& \quad \left( (a + i b - c) \left( 4c + 2b (-i + \tan[d + ex]) - 4i (a \tan[d + ex] + \sqrt{a + i b - c} \sqrt{c + \tan[d + ex] (b + a \tan[d + ex])}) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

**Problem 14: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[d + ex]}{(a + b \cot[d + ex] + c \cot[d + ex]^2)^{3/2}} dx$$

Optimal (type 3, 635 leaves, 7 steps):

$$\begin{aligned}
& - \left( \left( \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \right. \\
& \quad \left. \left. \text{ArcTanh} \left[ \left( b^2 - (a-c) \left( a-c + \sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c - \sqrt{a^2+b^2-2ac+c^2} \right) \cot[d+ex] \right) \right] / \right. \right. \\
& \quad \left. \left. \left( \sqrt{2} \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2} \right) \right] \right) / \\
& \quad \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{3/2} e \right) + \left( \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\
& \quad \left. \text{ArcTanh} \left[ \left( b^2 - (a-c) \left( a-c - \sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c + \sqrt{a^2+b^2-2ac+c^2} \right) \cot[d+ex] \right) \right] / \right. \\
& \quad \left. \left( \sqrt{2} \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2} \right) \right] \right) / \\
& \quad \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{3/2} e \right) - \frac{2(a(b^2-2(a-c)c)+bc(a+c)\cot[d+ex])}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2}}
\end{aligned}$$

Result (type 3, 3075 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{\frac{-a-c+a\cos[2(d+ex)]-c\cos[2(d+ex)]-b\sin[2(d+ex)]}{-1+\cos[2(d+ex)]}} \left( -\frac{2a(-b^2+2ac-2c^2)}{(a-c)(a-ib-c)(a+ib-c)(-b^2+4ac)} - \right. \\
& \quad \left. (2(-2ab^2c+4a^2c^2-4ac^3-ab^3\sin[2(d+ex)]+3a^2bc\sin[2(d+ex)]-2abc^2\sin[2(d+ex)]-bc^3\sin[2(d+ex)])) / \right. \\
& \quad \left. ((a-c)(a-ib-c)(a+ib-c)(-b^2+4ac)(-a-c+a\cos[2(d+ex)]-c\cos[2(d+ex)]-b\sin[2(d+ex)])) \right) + \\
& \quad \left( \sqrt{a+b\cot[d+ex]+c\cot[d+ex]^2} \left( -\frac{\text{Log} \left[ \frac{-4c-4ia\tan[d+ex]-2b(i+\tan[d+ex])+4i\sqrt{a-ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex])}{\sqrt{a-ib-c}(a+ib-c)(-i+\tan[d+ex])}}{(a-ib-c)^{3/2}} \right) + \right. \right. \\
& \quad \left. \left. \frac{\text{Log} \left[ \frac{4c+2b(-i+\tan[d+ex])-4i(a\tan[d+ex]+\sqrt{a+ib-c}\sqrt{c+\tan[d+ex]}(b+a\tan[d+ex]))}{(a-ib-c)\sqrt{a+ib-c}(i+\tan[d+ex])}}{(a+ib-c)^{3/2}} \right] \right) \right)
\end{aligned}$$

$$\left( \frac{b \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{(a - ib - c)(a + ib - c)(-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} + \right.$$

$$\frac{b \cos[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{(a - ib - c)(a + ib - c)(-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} -$$

$$\frac{a \sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{(a - ib - c)(a + ib - c)(-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} +$$

$$\left. \frac{c \sin[2(d+ex)] \sqrt{-\frac{a}{-1+\cos[2(d+ex)]} - \frac{c}{-1+\cos[2(d+ex)]} + \frac{a \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{-1+\cos[2(d+ex)]} - \frac{b \sin[2(d+ex)]}{-1+\cos[2(d+ex)]}}}{(a - ib - c)(a + ib - c)(-a - c + a \cos[2(d+ex)] - c \cos[2(d+ex)] - b \sin[2(d+ex)])} \right)$$

$$\left. \tan[d+ex] \right) / \left( 2e \sqrt{c + b \tan[d+ex] + a \tan[d+ex]^2} \right)$$

$$\left( \left( \left( \sqrt{a + b \cot[d+ex] + c \cot[d+ex]^2} \right) \left( \frac{\log \left[ \frac{-4c - 4ia \tan[d+ex] - 2b(i + \tan[d+ex]) + 4i \sqrt{a - ib - c} \sqrt{c + \tan[d+ex] (b + a \tan[d+ex])}}{\sqrt{a - ib - c} (a + ib - c) (-i + \tan[d+ex])} \right]}{(a - ib - c)^{3/2}} \right) + \right.$$

$$\left. \frac{\log \left[ \frac{4c + 2b(-i + \tan[d+ex]) - 4i(a \tan[d+ex] + \sqrt{a + ib - c} \sqrt{c + \tan[d+ex] (b + a \tan[d+ex])})}{(a - ib - c) \sqrt{a + ib - c} (i + \tan[d+ex])} \right]}{(a + ib - c)^{3/2}} \right) \tan[d+ex]$$

$$\left( b \sec[d+ex]^2 + 2a \sec[d+ex]^2 \tan[d+ex] \right) / \left( 4(c + b \tan[d+ex] + a \tan[d+ex]^2)^{3/2} \right) +$$

$$\begin{aligned}
& \left( \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \left( - \frac{\operatorname{Log} \left[ \frac{-4 c - 4 i a \tan [d + e x] - 2 b (i + \tan [d + e x]) + 4 i \sqrt{a - i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}}{\sqrt{a - i b - c} (a + i b - c) (-i + \tan [d + e x])} \right]}{(a - i b - c)^{3/2}} + \right. \right. \\
& \left. \left. \frac{\operatorname{Log} \left[ \frac{4 c + 2 b (-i + \tan [d + e x]) - 4 i (a \tan [d + e x] + \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])})}{(a - i b - c) \sqrt{a + i b - c} (i + \tan [d + e x])} \right]}{(a + i b - c)^{3/2}} \right) \operatorname{Sec} [d + e x]^2 \right) / \left( 2 \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2} \right) + \\
& \left( -b \operatorname{Csc} [d + e x]^2 - 2 c \cot [d + e x] \operatorname{Csc} [d + e x]^2 \right) \left( - \frac{\operatorname{Log} \left[ \frac{-4 c - 4 i a \tan [d + e x] - 2 b (i + \tan [d + e x]) + 4 i \sqrt{a - i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}}{\sqrt{a - i b - c} (a + i b - c) (-i + \tan [d + e x])} \right]}{(a - i b - c)^{3/2}} + \right. \\
& \left. \frac{\operatorname{Log} \left[ \frac{4 c + 2 b (-i + \tan [d + e x]) - 4 i (a \tan [d + e x] + \sqrt{a + i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])})}{(a - i b - c) \sqrt{a + i b - c} (i + \tan [d + e x])} \right]}{(a + i b - c)^{3/2}} \right) \operatorname{Tan} [d + e x] \right) / \\
& \left( 4 \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2} \right) + \frac{1}{2 \sqrt{c + b \tan [d + e x] + a \tan [d + e x]^2}} \\
& \sqrt{a + b \cot [d + e x] + c \cot [d + e x]^2} \operatorname{Tan} [d + e x] \left( - \left( \left( (a + i b - c) (-i + \tan [d + e x]) \right. \right. \right. \\
& \left. \left. \left( \left( -4 i a \operatorname{Sec} [d + e x]^2 - 2 b \operatorname{Sec} [d + e x]^2 + \frac{2 i \sqrt{a - i b - c} (a \operatorname{Sec} [d + e x]^2 \operatorname{Tan} [d + e x] + \operatorname{Sec} [d + e x]^2 (b + a \tan [d + e x]))}{\sqrt{c + \tan [d + e x] (b + a \tan [d + e x])}} \right) \right) / \right. \right. \\
& \left. \left. \left( \sqrt{a - i b - c} (a + i b - c) (-i + \tan [d + e x]) \right) - \left( \operatorname{Sec} [d + e x]^2 (-4 c - 4 i a \tan [d + e x] - 2 b (i + \tan [d + e x]) + \right. \right. \right. \\
& \left. \left. \left. 4 i \sqrt{a - i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right) / \left( \sqrt{a - i b - c} (a + i b - c) (-i + \tan [d + e x])^2 \right) \right) \right) / \\
& \left( (a - i b - c) \left( -4 c - 4 i a \tan [d + e x] - 2 b (i + \tan [d + e x]) + 4 i \sqrt{a - i b - c} \sqrt{c + \tan [d + e x] (b + a \tan [d + e x])} \right) \right) \right) +
\end{aligned}$$

$$\left( (a - i b - c) (i + \tan[d + ex]) \left( \left( 2b \sec[d + ex]^2 - 4i \left( a \sec[d + ex]^2 + \frac{\sqrt{a + i b - c} (a \sec[d + ex]^2 \tan[d + ex] + \sec[d + ex]^2 (b + a \tan[d + ex]))}{2\sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])} \right) \right) \right) / \right. \\
\left. \left( (a - i b - c) \sqrt{a + i b - c} (i + \tan[d + ex]) \right) - \left( \sec[d + ex]^2 (4c + 2b(-i + \tan[d + ex])) - 4i (a \tan[d + ex] + \sqrt{a + i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right) / \left. \left( (a - i b - c) \sqrt{a + i b - c} (i + \tan[d + ex])^2 \right) \right) / \\
\left( (a + i b - c) (4c + 2b(-i + \tan[d + ex])) - 4i (a \tan[d + ex] + \sqrt{a + i b - c} \sqrt{c + \tan[d + ex]} (b + a \tan[d + ex])) \right) \right) \right)$$

**Problem 15: Humongous result has more than 200000 leaves.**

$$\int \frac{\tan[d + ex]}{(a + b \cot[d + ex] + c \cot[d + ex]^2)^{3/2}} dx$$

Optimal (type 3, 749 leaves, 13 steps):



$$\begin{aligned}
& \frac{\text{ArcTanh}\left[\frac{2a+b\text{Cot}[d+ex]}{2\sqrt{a}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{a^{3/2}e} + \left(\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\right. \\
& \left.\text{ArcTanh}\left[\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\text{Cot}[d+ex]\right)\right]/\right. \\
& \left.\left(\sqrt{2}\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}\right)\right)/ \\
& \left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e\right) - \left(\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\right. \\
& \left.\text{ArcTanh}\left[\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\text{Cot}[d+ex]\right)\right]/\right. \\
& \left.\left(\sqrt{2}\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}\right)\right)/ \\
& \left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e\right) - \frac{2\left(b^2-2ac+bc\text{Cot}[d+ex]\right)}{a\left(b^2-4ac\right)e\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}} + \\
& \frac{2\left(a\left(b^2-2(a-c)c\right)+bc(a+c)\text{Cot}[d+ex]\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}
\end{aligned}$$

Result (type ?, 558961 leaves): Display of huge result suppressed!

**Problem 16: Humongous result has more than 200000 leaves.**

$$\int \frac{\text{Tan}[d+ex]^3}{\left(a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2\right)^{3/2}} dx$$

Optimal (type 3, 1008 leaves, 18 steps):

$$\begin{aligned}
& - \frac{\text{ArcTanh}\left[\frac{2a+b\text{Cot}[d+ex]}{2\sqrt{a}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{a^{3/2}e} + \frac{3(5b^2-4ac)\text{ArcTanh}\left[\frac{2a+b\text{Cot}[d+ex]}{2\sqrt{a}\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}}\right]}{8a^{7/2}e} \\
& \left( \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2} + (a-c)\sqrt{a^2+b^2-2ac+c^2} \right. \\
& \quad \left. \text{ArcTanh}\left[\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\text{Cot}[d+ex]\right)\right] \right) / \\
& \quad \left( \sqrt{2}\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2} + (a-c)\sqrt{a^2+b^2-2ac+c^2} \sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2} \right) \Big) / \\
& \left( \sqrt{2}(a^2+b^2-2ac+c^2)^{3/2}e \right) + \left( \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2} - (a-c)\sqrt{a^2+b^2-2ac+c^2} \right. \\
& \quad \left. \text{ArcTanh}\left[\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\text{Cot}[d+ex]\right)\right] \right) / \\
& \quad \left( \sqrt{2}\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2} - (a-c)\sqrt{a^2+b^2-2ac+c^2} \sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2} \right) \Big) / \\
& \left( \sqrt{2}(a^2+b^2-2ac+c^2)^{3/2}e \right) + \frac{2(b^2-2ac+bc\text{Cot}[d+ex])}{a(b^2-4ac)e\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}} - \\
& \frac{2(a(b^2-2(a-c)c)+bc(a+c)\text{Cot}[d+ex])}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}} - \\
& \frac{b(15b^2-52ac)\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}\text{Tan}[d+ex]}{4a^3(b^2-4ac)e} - \\
& \frac{2(b^2-2ac+bc\text{Cot}[d+ex])\text{Tan}[d+ex]^2}{a(b^2-4ac)e\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}} + \\
& \frac{(5b^2-12ac)\sqrt{a+b\text{Cot}[d+ex]+c\text{Cot}[d+ex]^2}\text{Tan}[d+ex]^2}{2a^2(b^2-4ac)e}
\end{aligned}$$

Result (type ?, 930953 leaves): Display of huge result suppressed!

### Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d + e x]^5}{\sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}} dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c) \text{Cot}[d + e x]^2}{2\sqrt{a - b + c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2\sqrt{a - b + c} e} + \frac{(b + 2c) \text{ArcTanh}\left[\frac{b + 2c \text{Cot}[d + e x]^2}{2\sqrt{c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{4c^{3/2} e} - \frac{\sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}{2ce}$$

Result (type 3, 2952 leaves):

$$\frac{\sqrt{\frac{3a + b + 3c - 4a \cos[2(d + e x)] + 4c \cos[2(d + e x)] + a \cos[4(d + e x)] - b \cos[4(d + e x)] + c \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]}}}{2ce} - \left( (b + 2c) \log[\tan[d + e x]^2] - \frac{2c^{3/2} \log[1 + \tan[d + e x]^2]}{\sqrt{a - b + c}} - b \log[2c + b \tan[d + e x]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + e x]^2 + a \tan[d + e x]^4}] - 2c \log[2c + b \tan[d + e x]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + e x]^2 + a \tan[d + e x]^4}] + \frac{1}{\sqrt{a - b + c}} \right) - 2c^{3/2} \log\left[b(-1 + \tan[d + e x]^2) + 2\left(c - a \tan[d + e x]^2 + \sqrt{a - b + c} \sqrt{c + b \tan[d + e x]^2 + a \tan[d + e x]^4}\right)\right] - \left( \left( 2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{b}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{3c}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} - \frac{4a \cos[2(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{4c \cos[2(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{a \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} - \frac{b \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{c \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} \right) \sin[2(d + e x)] \right) / \left( (3a + b + 3c - 4a \cos[2(d + e x)] + 4c \cos[2(d + e x)] + a \cos[4(d + e x)] - b \cos[4(d + e x)] + c \cos[4(d + e x)]) \right) - \left( 2b \sqrt{\left( \frac{3a}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{b}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{3c}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} - \frac{4a \cos[2(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{4c \cos[2(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{a \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} - \frac{b \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} + \frac{c \cos[4(d + e x)]}{3 - 4 \cos[2(d + e x)] + \cos[4(d + e x)]} \right) \right)$$

$$\begin{aligned}
 & \left. \left( \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [2 (d+e x)] \right) / \\
 & \left( c (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) \right) - \\
 & \left( \sqrt{\left( \frac{3 a}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{b}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{3 c}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
 & \left. \left. \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
 & \left. \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \right) / \\
 & \left. (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) \right) \\
 & \left. \tan [d+e x]^2 \sqrt{a+\cot [d+e x]^4 (c+b \tan [d+e x]^2)} \right) / 4 \\
 & c^{3/2} \\
 & e \\
 & \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4} \\
 & \left( \frac{1}{8 c^{3/2} (c+b \tan [d+e x]^2+a \tan [d+e x]^4)^{3/2}} \right. \\
 & \left. \left( (b+2 c) \log [\tan [d+e x]^2] - \frac{2 c^{3/2} \log [1+\tan [d+e x]^2]}{\sqrt{a-b+c}} - b \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] - \right. \right. \\
 & \left. \left. 2 c \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] + \frac{1}{\sqrt{a-b+c}} \right. \right. \\
 & \left. \left. 2 c^{3/2} \log [b (-1+\tan [d+e x]^2)+2 (c-a \tan [d+e x]^2+\sqrt{a-b+c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4})] \right) \right) \\
 & \tan [d+e x]^2 (2 b \sec [d+e x]^2 \tan [d+e x]+4 a \sec [d+e x]^2 \tan [d+e x]^3) \sqrt{a+\cot [d+e x]^4 (c+b \tan [d+e x]^2)} - \\
 & \frac{1}{2 c^{3/2} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \left( (b+2 c) \log [\tan [d+e x]^2] - \frac{2 c^{3/2} \log [1+\tan [d+e x]^2]}{\sqrt{a-b+c}} - b \log [2 c+b \tan [d+e x]^2+ \right. \\
 & \left. 2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] - 2 c \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a-b+c}} 2 c^{3/2} \operatorname{Log}\left[b\left(-1+\operatorname{Tan}[d+e x]^2\right)+2\left(c-a \operatorname{Tan}[d+e x]^2+\sqrt{a-b+c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}\right)\right] \\
& \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] \sqrt{a+\operatorname{Cot}[d+e x]^4\left(c+b \operatorname{Tan}[d+e x]^2\right)} - \\
& \left(\left((b+2 c) \operatorname{Log}[\operatorname{Tan}[d+e x]^2]-\frac{2 c^{3/2} \operatorname{Log}\left[1+\operatorname{Tan}[d+e x]^2\right]}{\sqrt{a-b+c}}-b \operatorname{Log}\left[2 c+b \operatorname{Tan}[d+e x]^2+2 \sqrt{c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}\right]-\right.\right. \\
& \quad \left.2 c \operatorname{Log}\left[2 c+b \operatorname{Tan}[d+e x]^2+2 \sqrt{c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}\right]+\frac{1}{\sqrt{a-b+c}}\right. \\
& \quad \left.2 c^{3/2} \operatorname{Log}\left[b\left(-1+\operatorname{Tan}[d+e x]^2\right)+2\left(c-a \operatorname{Tan}[d+e x]^2+\sqrt{a-b+c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}\right)\right]\right) \\
& \operatorname{Tan}[d+e x]^2\left(2 b \operatorname{Cot}[d+e x] \operatorname{Csc}[d+e x]^2-4 \operatorname{Cot}[d+e x]^3 \operatorname{Csc}[d+e x]^2\left(c+b \operatorname{Tan}[d+e x]^2\right)\right) / \\
& \left(8 c^{3/2} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4} \sqrt{a+\operatorname{Cot}[d+e x]^4\left(c+b \operatorname{Tan}[d+e x]^2\right)}\right) - \\
& \frac{1}{4 c^{3/2} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}} \operatorname{Tan}[d+e x]^2 \sqrt{a+\operatorname{Cot}[d+e x]^4\left(c+b \operatorname{Tan}[d+e x]^2\right)}\left(2(b+2 c) \operatorname{Csc}[d+e x] \operatorname{Sec}[d+e x]-\right. \\
& \frac{4 c^{3/2} \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]}{\sqrt{a-b+c}\left(1+\operatorname{Tan}[d+e x]^2\right)}-\frac{b\left(2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+\frac{\sqrt{c}\left(2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+4 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]^3\right)}{\sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}}\right)}{2 c+b \operatorname{Tan}[d+e x]^2+2 \sqrt{c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}} - \\
& \frac{2 c\left(2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+\frac{\sqrt{c}\left(2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+4 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]^3\right)}{\sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}}\right)}{2 c+b \operatorname{Tan}[d+e x]^2+2 \sqrt{c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}}+\left(2 c^{3/2}\left(2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+2\right.\right. \\
& \quad \left.\left.-2 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+\frac{\sqrt{a-b+c}\left(2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+4 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]^3\right)}{2 \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}}\right)\right) / \\
& \left.\left.\left.\left.\left(\sqrt{a-b+c}\left(b\left(-1+\operatorname{Tan}[d+e x]^2\right)+2\left(c-a \operatorname{Tan}[d+e x]^2+\sqrt{a-b+c} \sqrt{c+b \operatorname{Tan}[d+e x]^2+a \operatorname{Tan}[d+e x]^4}\right)\right)\right)\right)\right)\right)\right)
\end{aligned}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d + e x]^3}{\sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}} dx$$

Optimal (type 3, 141 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c) \text{Cot}[d + e x]^2}{2\sqrt{a - b + c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2\sqrt{a - b + c} e} - \frac{\text{ArcTanh}\left[\frac{b + 2c \text{Cot}[d + e x]^2}{2\sqrt{c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2\sqrt{c} e}$$

Result (type 3, 2161 leaves):

$$\left( \left( \frac{\text{Log}[\text{Tan}[d + e x]^2]}{\sqrt{c}} - \frac{\text{Log}[1 + \text{Tan}[d + e x]^2]}{\sqrt{a - b + c}} - \frac{\text{Log}[2c + b \text{Tan}[d + e x]^2 + 2\sqrt{c} \sqrt{c + \text{Tan}[d + e x]^2 (b + a \text{Tan}[d + e x]^2)}]}{\sqrt{c}} \right) + \frac{1}{\sqrt{a - b + c}} \text{Log}[b(-1 + \text{Tan}[d + e x]^2) + 2(c - a \text{Tan}[d + e x]^2 + \sqrt{a - b + c} \sqrt{c + \text{Tan}[d + e x]^2 (b + a \text{Tan}[d + e x]^2)})] \right) \left( 2 \sqrt{\left( \frac{3a}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{b}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{3c}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} - \frac{4a \text{Cos}[2(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{4c \text{Cos}[2(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{a \text{Cos}[4(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} - \frac{b \text{Cos}[4(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{c \text{Cos}[4(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} \right) \text{Sin}[2(d + e x)] \right) / (3a + b + 3c - 4a \text{Cos}[2(d + e x)] + 4c \text{Cos}[2(d + e x)] + a \text{Cos}[4(d + e x)] - b \text{Cos}[4(d + e x)] + c \text{Cos}[4(d + e x)]) + \left( \sqrt{\left( \frac{3a}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{b}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{3c}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} - \frac{4a \text{Cos}[2(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{4c \text{Cos}[2(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{a \text{Cos}[4(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} - \frac{b \text{Cos}[4(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} + \frac{c \text{Cos}[4(d + e x)]}{3 - 4 \text{Cos}[2(d + e x)] + \text{Cos}[4(d + e x)]} \right) \text{Sin}[4(d + e x)] \right) / (3a + b + 3c - 4a \text{Cos}[2(d + e x)] + 4c \text{Cos}[2(d + e x)] + a \text{Cos}[4(d + e x)] - b \text{Cos}[4(d + e x)] + c \text{Cos}[4(d + e x)]) \right) \left( \text{Tan}[d + e x]^2 \sqrt{a + \text{Cot}[d + e x]^4 (c + b \text{Tan}[d + e x]^2)} \right) / 2$$

$$\begin{aligned}
& e \\
& \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4} \\
& \left( \left( \left( \frac{\log [\tan [d + e x]^2]}{\sqrt{c}} - \frac{\log [1 + \tan [d + e x]^2]}{\sqrt{a - b + c}} - \frac{\log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + \tan [d + e x]^2 (b + a \tan [d + e x]^2)}}{\sqrt{c}} \right) + \right. \right. \\
& \quad \left. \left. \frac{1}{\sqrt{a - b + c}} \log [b (-1 + \tan [d + e x]^2) + 2 (c - a \tan [d + e x]^2 + \sqrt{a - b + c} \sqrt{c + \tan [d + e x]^2 (b + a \tan [d + e x]^2)})] \right) \right) \\
& \quad \tan [d + e x]^2 (2 b \sec [d + e x]^2 \tan [d + e x] + 4 a \sec [d + e x]^2 \tan [d + e x]^3) \\
& \quad \left. \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} \right) / \left( 4 (c + b \tan [d + e x]^2 + a \tan [d + e x]^4)^{3/2} \right) + \\
& \left( \left( \frac{\log [\tan [d + e x]^2]}{\sqrt{c}} - \frac{\log [1 + \tan [d + e x]^2]}{\sqrt{a - b + c}} - \frac{\log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + \tan [d + e x]^2 (b + a \tan [d + e x]^2)}}{\sqrt{c}} \right) + \right. \\
& \quad \left. \frac{1}{\sqrt{a - b + c}} \log [b (-1 + \tan [d + e x]^2) + 2 (c - a \tan [d + e x]^2 + \sqrt{a - b + c} \sqrt{c + \tan [d + e x]^2 (b + a \tan [d + e x]^2)})] \right) \\
& \quad \sec [d + e x]^2 \tan [d + e x] \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} \bigg/ \left( \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4} \right) + \\
& \left( \left( \frac{\log [\tan [d + e x]^2]}{\sqrt{c}} - \frac{\log [1 + \tan [d + e x]^2]}{\sqrt{a - b + c}} - \frac{\log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + \tan [d + e x]^2 (b + a \tan [d + e x]^2)}}{\sqrt{c}} \right) + \right. \\
& \quad \left. \frac{1}{\sqrt{a - b + c}} \log [b (-1 + \tan [d + e x]^2) + 2 (c - a \tan [d + e x]^2 + \sqrt{a - b + c} \sqrt{c + \tan [d + e x]^2 (b + a \tan [d + e x]^2)})] \right) \\
& \quad \tan [d + e x]^2 (2 b \cot [d + e x] \csc [d + e x]^2 - 4 \cot [d + e x]^3 \csc [d + e x]^2 (c + b \tan [d + e x]^2)) \bigg/ \\
& \quad \left( 4 \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4} \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} \right) + \\
& \quad \frac{1}{2 \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} \tan [d + e x]^2 \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} \left( \frac{2 \csc [d + e x] \sec [d + e x]}{\sqrt{c}} - \right.
\end{aligned}$$

$$\frac{2 \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]}{\sqrt{a-b+c} (1+\operatorname{Tan}[d+e x]^2)} - \frac{2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] + \frac{\sqrt{c} (2 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]^3 + 2 \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] (b+a \operatorname{Tan}[d+e x]^2))}{\sqrt{c+\operatorname{Tan}[d+e x]^2 (b+a \operatorname{Tan}[d+e x]^2)}}}{\sqrt{c} \left( 2 c + b \operatorname{Tan}[d+e x]^2 + 2 \sqrt{c} \sqrt{c + \operatorname{Tan}[d+e x]^2 (b+a \operatorname{Tan}[d+e x]^2)} \right)} +$$

$$\left( 2 b \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] + 2 \left( -2 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] + \left( \sqrt{a-b+c} (2 a \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]^3 + 2 \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] (b+a \operatorname{Tan}[d+e x]^2)) \right) \right) \right) / \left( 2 \sqrt{c + \operatorname{Tan}[d+e x]^2 (b+a \operatorname{Tan}[d+e x]^2)} \right) \Bigg) /$$

$$\left( \sqrt{a-b+c} \left( b (-1 + \operatorname{Tan}[d+e x]^2) + 2 \left( c - a \operatorname{Tan}[d+e x]^2 + \sqrt{a-b+c} \sqrt{c + \operatorname{Tan}[d+e x]^2 (b+a \operatorname{Tan}[d+e x]^2)} \right) \right) \right) \Bigg) \Bigg)$$

**Problem 19: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[d+e x]}{\sqrt{a+b \operatorname{Cot}[d+e x]^2+c \operatorname{Cot}[d+e x]^4}} dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2 a-b+(b-2 c) \operatorname{Cot}[d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \operatorname{Cot}[d+e x]^2+c \operatorname{Cot}[d+e x]^4}}\right]}{2 \sqrt{a-b+c} e}$$

Result (type 4, 84 039 leaves): Display of huge result suppressed!

**Problem 20: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[d+e x]}{\sqrt{a+b \operatorname{Cot}[d+e x]^2+c \operatorname{Cot}[d+e x]^4}} dx$$

Optimal (type 3, 142 leaves, 8 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2 a+b \operatorname{Cot}[d+e x]^2}{2 \sqrt{a} \sqrt{a+b \operatorname{Cot}[d+e x]^2+c \operatorname{Cot}[d+e x]^4}}\right]}{2 \sqrt{a} e} - \frac{\operatorname{ArcTanh}\left[\frac{2 a-b+(b-2 c) \operatorname{Cot}[d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \operatorname{Cot}[d+e x]^2+c \operatorname{Cot}[d+e x]^4}}\right]}{2 \sqrt{a-b+c} e}$$

Result (type 4, 44 361 leaves): Display of huge result suppressed!



**Problem 21: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\tan [d+e x]^3}{\sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}} dx$$

Optimal (type 3, 249 leaves, 11 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2 a+b \cot [d+e x]^2}{2 \sqrt{a} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}\right]}{2 \sqrt{a} e} - \frac{b \operatorname{ArcTanh}\left[\frac{2 a+b \cot [d+e x]^2}{2 \sqrt{a} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}\right]}{4 a^{3/2} e} + \frac{\operatorname{ArcTanh}\left[\frac{2 a-b+(b-2 c) \cot [d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}\right]}{2 \sqrt{a-b+c} e} + \frac{\sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4} \tan [d+e x]^2}{2 a e}$$

Result (type 4, 124 484 leaves): Display of huge result suppressed!

**Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \cot [d+e x]^5 \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4} dx$$

Optimal (type 3, 270 leaves, 9 steps):

$$\frac{\sqrt{a-b+c} \operatorname{ArcTanh}\left[\frac{2 a-b+(b-2 c) \cot [d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}\right]}{2 e} - \frac{\left(b^3+2 b^2 c-4 b(a-2 c) c-8 c^2(a+2 c)\right) \operatorname{ArcTanh}\left[\frac{b+2 c \cot [d+e x]^2}{2 \sqrt{c} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}\right]}{32 c^{5/2} e} + \frac{\left((b-2 c)(b+4 c)+2 c(b+2 c) \cot [d+e x]^2\right) \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}{16 c^2 e} - \frac{\left(a+b \cot [d+e x]^2+c \cot [d+e x]^4\right)^{3/2}}{6 c e}$$

Result (type 3, 4238 leaves):

$$\frac{1}{e} \sqrt{\left(\left(3 a+b+3 c-4 a \cos [2(d+e x)]+4 c \cos [2(d+e x)]+a \cos [4(d+e x)]-b \cos [4(d+e x)]+c \cos [4(d+e x)]\right)\right) / \left(3-4 \cos [2(d+e x)]+\cos [4(d+e x)]\right)} \left(-\frac{-3 b^2+8 a c-8 b c+44 c^2}{48 c^2}+\frac{(-b+14 c) \operatorname{Csc}[d+e x]^2}{24 c}-\frac{1}{6} \operatorname{Csc}[d+e x]^4\right) + \left(\left(\left(b^3+2 b^2 c-4 b(a-2 c) c-8 c^2(a+2 c)\right) \operatorname{Log}[\tan [d+e x]^2]+16 c^{5/2} \sqrt{a-b+c} \operatorname{Log}[1+\tan [d+e x]^2]-\left(b^3+2 b^2 c-4 b(a-2 c) c-8 c^2(a+2 c)\right) \operatorname{Log}\left[2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+\tan [d+e x]^2}\left(b+a \tan [d+e x]^2\right)\right]-16 c^{5/2} \sqrt{a-b+c} \operatorname{Log}\left[b\left(-1+\tan [d+e x]^2\right)+2\left(c-a \tan [d+e x]^2+\sqrt{a-b+c} \sqrt{c+\tan [d+e x]^2}\left(b+a \tan [d+e x]^2\right)\right)\right]\right)$$

$$\begin{aligned}
& \left( \left( b^3 \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \Big) / \\
& \left( 4c^2(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right) - \\
& \left( a b \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \Big) / \\
& \left( c(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right) + \\
& \left( b^2 \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \Big) / \\
& \left( 2c(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right) - \\
& \left( 2c \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \Big) / \\
& \left( 3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)] \right) - \\
& \left( a \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \Big/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) + \\
& \left( b \sqrt{\left( \frac{3 a}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{b}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{3 c}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
& \left. \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \Big/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) - \\
& \left( c \sqrt{\left( \frac{3 a}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{b}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{3 c}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
& \left. \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \Big/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) \Big) \\
& \left. \tan [d+e x]^2 \sqrt{a+\cot [d+e x]^4 (c+b \tan [d+e x]^2)} \right) \Big/ 32 \\
& c^{5/2} \\
& e \\
& \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4} \\
& \left( -\frac{1}{64 c^{5/2} (c+b \tan [d+e x]^2+a \tan [d+e x]^4)^{3/2}} \right. \\
& \left. \left( (b^3+2 b^2 c-4 b (a-2 c) c-8 c^2 (a+2 c)) \log [\tan [d+e x]^2]+16 c^{5/2} \sqrt{a-b+c} \log [1+\tan [d+e x]^2] - \right. \right. \\
& \left. (b^3+2 b^2 c-4 b (a-2 c) c-8 c^2 (a+2 c)) \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+\tan [d+e x]^2 (b+a \tan [d+e x]^2)}] - \right. \\
& \left. \left. 16 c^{5/2} \sqrt{a-b+c} \log [b(-1+\tan [d+e x]^2)+2 (c-a \tan [d+e x]^2+\sqrt{a-b+c} \sqrt{c+\tan [d+e x]^2 (b+a \tan [d+e x]^2)})] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\tan[d+ex]^2 (2b \sec[d+ex]^2 \tan[d+ex] + 4a \sec[d+ex]^2 \tan[d+ex]^3) \sqrt{a + \cot[d+ex]^4 (c + b \tan[d+ex]^2)} +}{16c^{5/2} \sqrt{c + b \tan[d+ex]^2 + a \tan[d+ex]^4}} \left( (b^3 + 2b^2c - 4b(a-2c)c - 8c^2(a+2c)) \log[\tan[d+ex]^2] + 16c^{5/2} \sqrt{a-b+c} \log[1 + \right. \\
& \left. \tan[d+ex]^2] - (b^3 + 2b^2c - 4b(a-2c)c - 8c^2(a+2c)) \log[2c + b \tan[d+ex]^2 + 2\sqrt{c} \sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)}] \right) - \\
& 16c^{5/2} \sqrt{a-b+c} \log[b(-1 + \tan[d+ex]^2) + 2(c - a \tan[d+ex]^2 + \sqrt{a-b+c} \sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)})] \Big) \\
& \frac{\sec[d+ex]^2 \tan[d+ex] \sqrt{a + \cot[d+ex]^4 (c + b \tan[d+ex]^2)} +}{\left( (b^3 + 2b^2c - 4b(a-2c)c - 8c^2(a+2c)) \log[\tan[d+ex]^2] + 16c^{5/2} \sqrt{a-b+c} \log[1 + \tan[d+ex]^2] - \right.} \\
& \left. (b^3 + 2b^2c - 4b(a-2c)c - 8c^2(a+2c)) \log[2c + b \tan[d+ex]^2 + 2\sqrt{c} \sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)}] - \right. \\
& \left. 16c^{5/2} \sqrt{a-b+c} \log[b(-1 + \tan[d+ex]^2) + 2(c - a \tan[d+ex]^2 + \sqrt{a-b+c} \sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)})] \right) \Big) \\
& \tan[d+ex]^2 (2b \cot[d+ex] \csc[d+ex]^2 - 4 \cot[d+ex]^3 \csc[d+ex]^2 (c + b \tan[d+ex]^2)) \Big) / \\
& \left( 64c^{5/2} \sqrt{c + b \tan[d+ex]^2 + a \tan[d+ex]^4} \sqrt{a + \cot[d+ex]^4 (c + b \tan[d+ex]^2)} \right) + \frac{1}{32c^{5/2} \sqrt{c + b \tan[d+ex]^2 + a \tan[d+ex]^4}} \\
& \tan[d+ex]^2 \sqrt{a + \cot[d+ex]^4 (c + b \tan[d+ex]^2)} \left( 2(b^3 + 2b^2c - 4b(a-2c)c - 8c^2(a+2c)) \csc[d+ex] \sec[d+ex] + \right. \\
& \left. \frac{32c^{5/2} \sqrt{a-b+c} \sec[d+ex]^2 \tan[d+ex]}{1 + \tan[d+ex]^2} - (b^3 + 2b^2c - 4b(a-2c)c - 8c^2(a+2c)) \right. \\
& \left. \left( 2b \sec[d+ex]^2 \tan[d+ex] + \frac{\sqrt{c} (2a \sec[d+ex]^2 \tan[d+ex]^3 + 2 \sec[d+ex]^2 \tan[d+ex] (b + a \tan[d+ex]^2))}{\sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)}} \right) \right) / \\
& \left( 2c + b \tan[d+ex]^2 + 2\sqrt{c} \sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)} \right) - \\
& \left( 16c^{5/2} \sqrt{a-b+c} (2b \sec[d+ex]^2 \tan[d+ex] + 2(-2a \sec[d+ex]^2 \tan[d+ex] + (\sqrt{a-b+c} (2a \sec[d+ex]^2 \tan[d+ex]^3 + \right. \\
& \left. 2 \sec[d+ex]^2 \tan[d+ex] (b + a \tan[d+ex]^2)))) / (2\sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)}) \right) \Big) / \\
& \left( b(-1 + \tan[d+ex]^2) + 2(c - a \tan[d+ex]^2 + \sqrt{a-b+c} \sqrt{c + \tan[d+ex]^2 (b + a \tan[d+ex]^2)}) \right) \Big) \Big)
\end{aligned}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[d + e x]^3 \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4} dx$$

Optimal (type 3, 209 leaves, 8 steps):

$$\frac{\sqrt{a - b + c} \text{ArcTanh}\left[\frac{2a - b + (b - 2c) \text{Cot}[d + e x]^2}{2\sqrt{a - b + c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2e} +$$

$$\frac{(b^2 + 4bc - 4c(a + 2c)) \text{ArcTanh}\left[\frac{b + 2c \text{Cot}[d + e x]^2}{2\sqrt{c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{16c^{3/2}e} - \frac{(b - 4c + 2c \text{Cot}[d + e x]^2) \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}{8ce}$$

Result (type 3, 4379 leaves):

$$\frac{\sqrt{\frac{3a + b + 3c - 4a \cos[2(d + ex)] + 4c \cos[2(d + ex)] + a \cos[4(d + ex)] - b \cos[4(d + ex)] + c \cos[4(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]}} \left( \frac{-b + 6c}{8c} - \frac{1}{4} \text{Csc}[d + ex]^2 \right)}{e} +$$

$$\left( - (b^2 + 4bc - 4c(a + 2c)) \text{Log}[\text{Tan}[d + ex]^2] - \right.$$

$$8c^{3/2} \sqrt{a - b + c} \text{Log}[1 + \text{Tan}[d + ex]^2] + b^2 \text{Log}[2c + b \text{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \text{Tan}[d + ex]^2 + a \text{Tan}[d + ex]^4}] -$$

$$4ac \text{Log}[2c + b \text{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \text{Tan}[d + ex]^2 + a \text{Tan}[d + ex]^4}] +$$

$$4bc \text{Log}[2c + b \text{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \text{Tan}[d + ex]^2 + a \text{Tan}[d + ex]^4}] -$$

$$8c^2 \text{Log}[2c + b \text{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \text{Tan}[d + ex]^2 + a \text{Tan}[d + ex]^4}] +$$

$$8c^{3/2} \sqrt{a - b + c} \text{Log}\left[ b(-1 + \text{Tan}[d + ex]^2) + 2 \left( c - a \text{Tan}[d + ex]^2 + \sqrt{a - b + c} \sqrt{c + b \text{Tan}[d + ex]^2 + a \text{Tan}[d + ex]^4} \right) \right] \Big)$$

$$\left( - \left( \left( b^2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{b}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{3c}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} - \right. \right. \right. \right.$$

$$\frac{4a \cos[2(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{4c \cos[2(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{a \cos[4(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} -$$

$$\frac{b \cos[4(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{c \cos[4(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} \Big) \text{Sin}[2(d + ex)] \Big) /$$

$$(2c(3a + b + 3c - 4a \cos[2(d + ex)] + 4c \cos[2(d + ex)] + a \cos[4(d + ex)] - b \cos[4(d + ex)] + c \cos[4(d + ex)])) \Big) +$$

$$\left( 2c \sqrt{\left( \frac{3a}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{b}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{3c}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} - \right. \right. \right.$$

$$\begin{aligned}
& \left. \begin{aligned}
& \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \\
& \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \left. \right) \sin [2 (d+e x)] \Bigg/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) + \\
& \left( a \sqrt{\left( \frac{3 a}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{b}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{3 c}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
& \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \Bigg/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) - \\
& \left( b \sqrt{\left( \frac{3 a}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{b}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{3 c}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
& \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \Bigg/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) + \\
& \left( c \sqrt{\left( \frac{3 a}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{b}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{3 c}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \right. \right. \\
& \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sin [4 (d+e x)] \Bigg/ \\
& (3 a+b+3 c-4 a \cos [2 (d+e x)]+4 c \cos [2 (d+e x)]+a \cos [4 (d+e x)]-b \cos [4 (d+e x)]+c \cos [4 (d+e x)]) \Bigg)
\end{aligned}
\right) \Bigg/ \left( \tan [d+e x]^2 \sqrt{a+\cot [d+e x]^4 (c+b \tan [d+e x]^2)} \right) \Bigg/ 16
\end{aligned}$$

e

$$\begin{aligned}
& \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4} \\
& \left( \frac{1}{32 c^{3/2} (c + b \tan[d + ex]^2 + a \tan[d + ex]^4)^{3/2}} \left( - (b^2 + 4bc - 4c(a + 2c)) \operatorname{Log}[\tan[d + ex]^2] - \right. \right. \\
& \quad 8c^{3/2} \sqrt{a - b + c} \operatorname{Log}[1 + \tan[d + ex]^2] + b^2 \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - \\
& \quad 4ac \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + 4bc \operatorname{Log}[2c + b \tan[d + ex]^2 + \\
& \quad \left. 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - 8c^2 \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + \right. \\
& \quad \left. 8c^{3/2} \sqrt{a - b + c} \operatorname{Log}[b(-1 + \tan[d + ex]^2) + 2(c - a \tan[d + ex]^2 + \sqrt{a - b + c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4})] \right) \\
& \quad \tan[d + ex]^2 (2b \operatorname{Sec}[d + ex]^2 \tan[d + ex] + 4a \operatorname{Sec}[d + ex]^2 \tan[d + ex]^3) \sqrt{a + \operatorname{Cot}[d + ex]^4 (c + b \tan[d + ex]^2)} + \\
& \quad \frac{1}{8c^{3/2} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}} \left( - (b^2 + 4bc - 4c(a + 2c)) \operatorname{Log}[\tan[d + ex]^2] - 8c^{3/2} \sqrt{a - b + c} \operatorname{Log}[1 + \tan[d + ex]^2] + \right. \\
& \quad b^2 \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - \\
& \quad 4ac \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + 4bc \operatorname{Log}[2c + b \tan[d + ex]^2 + \\
& \quad \left. 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - 8c^2 \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + \right. \\
& \quad \left. 8c^{3/2} \sqrt{a - b + c} \operatorname{Log}[b(-1 + \tan[d + ex]^2) + 2(c - a \tan[d + ex]^2 + \sqrt{a - b + c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4})] \right) \\
& \quad \operatorname{Sec}[d + ex]^2 \tan[d + ex] \sqrt{a + \operatorname{Cot}[d + ex]^4 (c + b \tan[d + ex]^2)} + \left( \left( - (b^2 + 4bc - 4c(a + 2c)) \operatorname{Log}[\tan[d + ex]^2] - \right. \right. \\
& \quad 8c^{3/2} \sqrt{a - b + c} \operatorname{Log}[1 + \tan[d + ex]^2] + b^2 \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - \\
& \quad 4ac \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + 4bc \operatorname{Log}[2c + b \tan[d + ex]^2 + \\
& \quad \left. 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - 8c^2 \operatorname{Log}[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + \right. \\
& \quad \left. 8c^{3/2} \sqrt{a - b + c} \operatorname{Log}[b(-1 + \tan[d + ex]^2) + 2(c - a \tan[d + ex]^2 + \sqrt{a - b + c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4})] \right) \\
& \quad \tan[d + ex]^2 (2b \operatorname{Cot}[d + ex] \operatorname{Csc}[d + ex]^2 - 4 \operatorname{Cot}[d + ex]^3 \operatorname{Csc}[d + ex]^2 (c + b \tan[d + ex]^2)) \Big/ \\
& \quad \left( 32c^{3/2} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4} \sqrt{a + \operatorname{Cot}[d + ex]^4 (c + b \tan[d + ex]^2)} \right) + \\
& \quad \frac{1}{16c^{3/2} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}} \tan[d + ex]^2 \sqrt{a + \operatorname{Cot}[d + ex]^4 (c + b \tan[d + ex]^2)}
\end{aligned}$$

$$\begin{aligned}
& \left( -2 (b^2 + 4bc - 4c(a + 2c)) \operatorname{Csc}[d + ex] \operatorname{Sec}[d + ex] - \frac{16c^{3/2} \sqrt{a - b + c} \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex]}{1 + \operatorname{Tan}[d + ex]^2} + \right. \\
& \frac{b^2 \left( 2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \frac{\sqrt{c} (2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + 4a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex]^3)}{\sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} \right)}{2c + b \operatorname{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} - \\
& \frac{4ac \left( 2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \frac{\sqrt{c} (2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + 4a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex]^3)}{\sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} \right)}{2c + b \operatorname{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} + \\
& \frac{4bc \left( 2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \frac{\sqrt{c} (2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + 4a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex]^3)}{\sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} \right)}{2c + b \operatorname{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} - \\
& \frac{8c^2 \left( 2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \frac{\sqrt{c} (2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + 4a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex]^3)}{\sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} \right)}{2c + b \operatorname{Tan}[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} + \left( 8c^{3/2} \sqrt{a - b + c} \left( 2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + 2 \right. \right. \\
& \left. \left. \left( -2a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + \frac{\sqrt{a - b + c} (2b \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex] + 4a \operatorname{Sec}[d + ex]^2 \operatorname{Tan}[d + ex]^3)}{2\sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4}} \right) \right) \right) / \\
& \left. \left. \left. \left( b(-1 + \operatorname{Tan}[d + ex]^2) + 2 \left( c - a \operatorname{Tan}[d + ex]^2 + \sqrt{a - b + c} \sqrt{c + b \operatorname{Tan}[d + ex]^2 + a \operatorname{Tan}[d + ex]^4} \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 24: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[d + ex] \sqrt{a + b \operatorname{Cot}[d + ex]^2 + c \operatorname{Cot}[d + ex]^4} dx$$

Optimal (type 3, 179 leaves, 8 steps):

$$\frac{\sqrt{a - b + c} \operatorname{ArcTanh}\left[\frac{2a - b + (b - 2c) \operatorname{Cot}[d + ex]^2}{2\sqrt{a - b + c} \sqrt{a + b \operatorname{Cot}[d + ex]^2 + c \operatorname{Cot}[d + ex]^4}}\right]}{2e} - \frac{(b - 2c) \operatorname{ArcTanh}\left[\frac{b + 2c \operatorname{Cot}[d + ex]^2}{2\sqrt{c} \sqrt{a + b \operatorname{Cot}[d + ex]^2 + c \operatorname{Cot}[d + ex]^4}}\right]}{4\sqrt{c}e} - \frac{\sqrt{a + b \operatorname{Cot}[d + ex]^2 + c \operatorname{Cot}[d + ex]^4}}{2e}$$



Result (type 3, 3486 leaves):

$$\begin{aligned}
 & - \frac{\sqrt{\frac{3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]}}}{2e} + \left( \sqrt{a+b\cot[d+ex]^2+c\cot[d+ex]^4} \right. \\
 & \left. \left( 2\sqrt{c}\sqrt{a-b+c}\log[\sec[d+ex]^2] + (b-2c)\log[\tan[d+ex]^2] - b\log[2c+b\tan[d+ex]^2+2\sqrt{c}\sqrt{c+b\tan[d+ex]^2+a\tan[d+ex]^4}] + \right. \right. \\
 & \left. \left. 2c\log[2c+b\tan[d+ex]^2+2\sqrt{c}\sqrt{c+b\tan[d+ex]^2+a\tan[d+ex]^4}] - \right. \right. \\
 & \left. \left. 2\sqrt{c}\sqrt{a-b+c}\log[-b+(-2a+b)\tan[d+ex]^2+2\left(c+\sqrt{a-b+c}\sqrt{c+b\tan[d+ex]^2+a\tan[d+ex]^4}\right)] \right) \right) \\
 & \left( \left( 2a\sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \right. \\
 & \left. \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
 & \left. \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \right) / \\
 & \left. \left( 3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)] \right) - \right. \\
 & \left( 2c\sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
 & \left. \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
 & \left. \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \right) / \\
 & \left. \left( 3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)] \right) - \right. \\
 & \left( a\sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
 & \left. \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
 & \left. \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[4(d+ex)] \right) / \\
 & \left. \left( 3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)] \right) + \right. \\
 & \left. \left( b\sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{4a \cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c \cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \\
& \left. \frac{b \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[4(d+ex)] \Big/ \\
& (3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) - \\
& \left( c \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right.} \right. \\
& \left. \frac{4a \cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c \cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \left. \frac{b \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[4(d+ex)] \Big/ \\
& (3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \Big) \tan[d+ex]^2 \Big/ \\
& \left( 4\sqrt{c} e^{\sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}} \left( -\frac{1}{8\sqrt{c} (c+b \tan[d+ex]^2+a \tan[d+ex]^4)^{3/2}} \right. \right. \\
& \left. \sqrt{a+b \cot[d+ex]^2+c \cot[d+ex]^4} \left( 2\sqrt{c} \sqrt{a-b+c} \log[\sec[d+ex]^2] + (b-2c) \log[\tan[d+ex]^2] - b \log[2c+b \tan[d+ex]^2] + \right. \right. \\
& \left. \left. 2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4} \right) + 2c \log[2c+b \tan[d+ex]^2+2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}] - \right. \\
& \left. \left. 2\sqrt{c} \sqrt{a-b+c} \log[-b+(-2a+b) \tan[d+ex]^2+2(c+\sqrt{a-b+c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4})] \right) \right) \\
& \tan[d+ex]^2 (2b \sec[d+ex]^2 \tan[d+ex] + 4a \sec[d+ex]^2 \tan[d+ex]^3) + \\
& \frac{1}{2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}} \sqrt{a+b \cot[d+ex]^2+c \cot[d+ex]^4} \left( 2\sqrt{c} \sqrt{a-b+c} \log[\sec[d+ex]^2] + \right. \\
& (b-2c) \log[\tan[d+ex]^2] - b \log[2c+b \tan[d+ex]^2+2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}] + \\
& 2c \log[2c+b \tan[d+ex]^2+2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}] - \\
& \left. \left. 2\sqrt{c} \sqrt{a-b+c} \log[-b+(-2a+b) \tan[d+ex]^2+2(c+\sqrt{a-b+c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4})] \right) \right) \sec[d+ex]^2 \tan[d+ex] + \\
& \left( (-2b \cot[d+ex] \csc[d+ex]^2 - 4c \cot[d+ex]^3 \csc[d+ex]^2) \left( 2\sqrt{c} \sqrt{a-b+c} \log[\sec[d+ex]^2] + \right. \right. \\
& \left. (b-2c) \log[\tan[d+ex]^2] - b \log[2c+b \tan[d+ex]^2+2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}] + \right. \\
& \left. \left. 2c \log[2c+b \tan[d+ex]^2+2\sqrt{c} \sqrt{c+b \tan[d+ex]^2+a \tan[d+ex]^4}] - \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{4 a \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{4 c \cos [2 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{a \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} - \\
& \left. \frac{b \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} + \frac{c \cos [4 (d+e x)]}{3-4 \cos [2 (d+e x)]+\cos [4 (d+e x)]} \right) \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4} \\
& \left( -\sqrt{a-b+c} \log [\sec [d+e x]^2] + \sqrt{c} \log [\tan [d+e x]^2] + \sqrt{a} \log [b+2 a \tan [d+e x]^2+2 \sqrt{a} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] - \right. \\
& \left. \sqrt{c} \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] + \right. \\
& \left. \sqrt{a-b+c} \log [-b+(-2 a+b) \tan [d+e x]^2+2\left(c+\sqrt{a-b+c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}\right)] \right) \tan [d+e x]^3 \Big/ \\
& \left( 2 e \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4} \left( -\frac{1}{4\left(c+b \tan [d+e x]^2+a \tan [d+e x]^4\right)^{3 / 2}} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4} \right. \right. \\
& \left. \left. \left( -\sqrt{a-b+c} \log [\sec [d+e x]^2] + \sqrt{c} \log [\tan [d+e x]^2] + \sqrt{a} \log [b+2 a \tan [d+e x]^2+2 \sqrt{a} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] - \right. \right. \right. \\
& \left. \left. \left. \sqrt{c} \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] + \right. \right. \right. \\
& \left. \left. \left. \sqrt{a-b+c} \log [-b+(-2 a+b) \tan [d+e x]^2+2\left(c+\sqrt{a-b+c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}\right)] \right) \right) \right) \\
& \tan [d+e x]^2\left(2 b \sec [d+e x]^2 \tan [d+e x]+4 a \sec [d+e x]^2 \tan [d+e x]^3\right)+\frac{1}{\sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \\
& \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}\left(-\sqrt{a-b+c} \log [\sec [d+e x]^2] + \sqrt{c} \log [\tan [d+e x]^2] + \sqrt{a} \log [b+2 a \tan [d+e x]^2+ \right. \\
& \left. 2 \sqrt{a} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] - \sqrt{c} \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] + \right. \\
& \left. \sqrt{a-b+c} \log [-b+(-2 a+b) \tan [d+e x]^2+2\left(c+\sqrt{a-b+c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}\right)]\right) \sec [d+e x]^2 \tan [d+e x]+ \\
& \left( (-2 b \cot [d+e x] \csc [d+e x]^2-4 c \cot [d+e x]^3 \csc [d+e x]^2)\left(-\sqrt{a-b+c} \log [\sec [d+e x]^2] + \sqrt{c} \log [\tan [d+e x]^2] + \right. \right. \\
& \left. \left. \sqrt{a} \log [b+2 a \tan [d+e x]^2+2 \sqrt{a} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] - \right. \right. \\
& \left. \left. \sqrt{c} \log [2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}] + \right. \right. \\
& \left. \left. \sqrt{a-b+c} \log [-b+(-2 a+b) \tan [d+e x]^2+2\left(c+\sqrt{a-b+c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}\right)]\right) \right) \tan [d+e x]^2 \Big/ \\
& \left( 4 \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4} \right) + \frac{1}{2 \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4} \tan [d + e x]^2 \left( 2 \sqrt{c} \csc [d + e x] \sec [d + e x] - 2 \sqrt{a - b + c} \tan [d + e x] + \right. \\
& \left. \frac{\sqrt{a} \left( 4 a \sec [d + e x]^2 \tan [d + e x] + \frac{\sqrt{a} (2 b \sec [d + e x]^2 \tan [d + e x] + 4 a \sec [d + e x]^2 \tan [d + e x]^3)}{\sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} \right)}{b + 2 a \tan [d + e x]^2 + 2 \sqrt{a} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} - \right. \\
& \left. \frac{\sqrt{c} \left( 2 b \sec [d + e x]^2 \tan [d + e x] + \frac{\sqrt{c} (2 b \sec [d + e x]^2 \tan [d + e x] + 4 a \sec [d + e x]^2 \tan [d + e x]^3)}{\sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} \right)}{2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} + \right. \\
& \left. \left( \sqrt{a - b + c} \left( 2 (-2 a + b) \sec [d + e x]^2 \tan [d + e x] + \frac{\sqrt{a - b + c} (2 b \sec [d + e x]^2 \tan [d + e x] + 4 a \sec [d + e x]^2 \tan [d + e x]^3)}{\sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} \right) \right) \right) / \\
& \left. \left( -b + (-2 a + b) \tan [d + e x]^2 + 2 \left( c + \sqrt{a - b + c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4} \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 26: Humongous result has more than 200000 leaves.

$$\int \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4} \tan [d + e x]^3 dx$$

Optimal (type 3, 435 leaves, 22 steps):

$$\begin{aligned}
& - \frac{\sqrt{a} \operatorname{ArcTanh} \left[ \frac{2 a + b \cot [d + e x]^2}{2 \sqrt{a} \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4}} \right]}{2 e} + \frac{b \operatorname{ArcTanh} \left[ \frac{2 a + b \cot [d + e x]^2}{2 \sqrt{a} \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4}} \right]}{4 \sqrt{a} e} + \frac{\sqrt{a - b + c} \operatorname{ArcTanh} \left[ \frac{2 a - b + (b - 2 c) \cot [d + e x]^2}{2 \sqrt{a - b + c} \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4}} \right]}{2 e} + \\
& \frac{b \operatorname{ArcTanh} \left[ \frac{b + 2 c \cot [d + e x]^2}{2 \sqrt{c} \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4}} \right]}{4 \sqrt{c} e} - \frac{(b - 2 c) \operatorname{ArcTanh} \left[ \frac{b + 2 c \cot [d + e x]^2}{2 \sqrt{c} \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4}} \right]}{4 \sqrt{c} e} - \\
& \frac{\sqrt{c} \operatorname{ArcTanh} \left[ \frac{b + 2 c \cot [d + e x]^2}{2 \sqrt{c} \sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4}} \right]}{2 e} + \frac{\sqrt{a + b \cot [d + e x]^2 + c \cot [d + e x]^4} \tan [d + e x]^2}{2 e}
\end{aligned}$$

Result (type ?, 215 131 leaves): Display of huge result suppressed!

### Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d + e x]^7}{(a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 236 leaves, 8 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c) \text{Cot}[d + e x]^2}{2\sqrt{a - b + c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2(a - b + c)^{3/2} e} - \frac{\text{ArcTanh}\left[\frac{b + 2c \text{Cot}[d + e x]^2}{2\sqrt{c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right] - \frac{a(b^2 - a(b + 2c)) + (b^3 + 2a^2c - ab(b + 3c)) \text{Cot}[d + e x]^2}{c(a - b + c)(b^2 - 4ac) e \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}}{2c^{3/2} e}}$$

Result (type 3, 3921 leaves):

$$\frac{1}{e} \sqrt{\left( (3a + b + 3c - 4a \cos[2(d + ex)] + 4c \cos[2(d + ex)] + a \cos[4(d + ex)] - b \cos[4(d + ex)] + c \cos[4(d + ex)]) \right) / \left( (3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]) \right)} - \frac{a^2 b - 2ab^2 + b^3 + 4a^2 c - 3abc}{c(a - b + c)^2 (-b^2 + 4ac)} - \left( 4(-2a^3 + a^2 b + a b^2 - b^3 - 2a^2 c + 3abc + 2a^3 \cos[2(d + ex)] - 3a^2 b \cos[2(d + ex)] + 3ab^2 \cos[2(d + ex)] - b^3 \cos[2(d + ex)] - 6a^2 c \cos[2(d + ex)] + 3abc \cos[2(d + ex)]) \right) / \left( (a - b + c)^2 (-b^2 + 4ac) \left( 3a + b + 3c - 4a \cos[2(d + ex)] + 4c \cos[2(d + ex)] + a \cos[4(d + ex)] - b \cos[4(d + ex)] + c \cos[4(d + ex)] \right) \right) + \left( \left( (a - b + c) \log[\tan[d + ex]^2] - \frac{c^{3/2} \log[1 + \tan[d + ex]^2]}{\sqrt{a - b + c}} - a \log[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] \right) + b \log[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] - c \log[2c + b \tan[d + ex]^2 + 2\sqrt{c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4}] + \frac{1}{\sqrt{a - b + c}} c^{3/2} \log[b(-1 + \tan[d + ex]^2) + 2(c - a \tan[d + ex]^2 + \sqrt{a - b + c} \sqrt{c + b \tan[d + ex]^2 + a \tan[d + ex]^4})] \right) \left( 2 \sqrt{\left( \frac{3a}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{b}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{3c}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} - \frac{4a \cos[2(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{4c \cos[2(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} + \frac{a \cos[4(d + ex)]}{3 - 4 \cos[2(d + ex)] + \cos[4(d + ex)]} \right)} \right)$$

$$\begin{aligned}
& \left. \frac{\frac{b \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c \cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]}}{\left( (a-b+c)(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right)} \right) \Big/ \\
& \left( 4a \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \Big/ \\
& \quad \left( c(a-b+c)(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right) - \\
& \left( 4b \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[2(d+ex)] \Big/ \\
& \quad \left( c(a-b+c)(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right) + \\
& \left( \sqrt{\left( \frac{3a}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{b}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{3c}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \right. \\
& \quad \left. \frac{4a\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{4c\cos[2(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{a\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} - \right. \\
& \quad \left. \frac{b\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} + \frac{c\cos[4(d+ex)]}{3-4\cos[2(d+ex)]+\cos[4(d+ex)]} \right) \sin[4(d+ex)] \Big/ \\
& \quad \left( (a-b+c)(3a+b+3c-4a\cos[2(d+ex)]+4c\cos[2(d+ex)]+a\cos[4(d+ex)]-b\cos[4(d+ex)]+c\cos[4(d+ex)]) \right) \Big) \\
& \tan[d+ex]^2 \sqrt{a + \cot[d+ex]^4 (c + b \tan[d+ex]^2)} \Big/ \left( 2 \right. \\
& \left. \begin{array}{l} c^{3/2} \\ (a-b+c) \\ e \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4} \\
& \left( - \frac{1}{4 c^{3/2} (a - b + c) (c + b \tan [d + e x]^2 + a \tan [d + e x]^4)^{3/2}} \right. \\
& \left( (a - b + c) \log [\tan [d + e x]^2] - \frac{c^{3/2} \log [1 + \tan [d + e x]^2]}{\sqrt{a - b + c}} - a \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] + \right. \\
& \quad b \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] - \\
& \quad c \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] + \frac{1}{\sqrt{a - b + c}} \\
& \quad \left. \left. c^{3/2} \log [b (-1 + \tan [d + e x]^2) + 2 (c - a \tan [d + e x]^2 + \sqrt{a - b + c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4})] \right) \right) \\
& \frac{\tan [d + e x]^2 (2 b \sec [d + e x]^2 \tan [d + e x] + 4 a \sec [d + e x]^2 \tan [d + e x]^3) \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} +}{c^{3/2} (a - b + c) \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}} \left( (a - b + c) \log [\tan [d + e x]^2] - \frac{c^{3/2} \log [1 + \tan [d + e x]^2]}{\sqrt{a - b + c}} - \right. \\
& \quad a \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] + b \log [2 c + b \tan [d + e x]^2 + \\
& \quad \left. 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] - c \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] + \right. \\
& \quad \left. \frac{1}{\sqrt{a - b + c}} c^{3/2} \log [b (-1 + \tan [d + e x]^2) + 2 (c - a \tan [d + e x]^2 + \sqrt{a - b + c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4})] \right) \\
& \sec [d + e x]^2 \tan [d + e x] \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} + \\
& \left( \left( (a - b + c) \log [\tan [d + e x]^2] - \frac{c^{3/2} \log [1 + \tan [d + e x]^2]}{\sqrt{a - b + c}} - a \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] + \right. \right. \\
& \quad b \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] - \\
& \quad c \log [2 c + b \tan [d + e x]^2 + 2 \sqrt{c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4}] + \frac{1}{\sqrt{a - b + c}} \\
& \quad \left. \left. c^{3/2} \log [b (-1 + \tan [d + e x]^2) + 2 (c - a \tan [d + e x]^2 + \sqrt{a - b + c} \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4})] \right) \right) \\
& \frac{\tan [d + e x]^2 (2 b \cot [d + e x] \csc [d + e x]^2 - 4 \cot [d + e x]^3 \csc [d + e x]^2 (c + b \tan [d + e x]^2))}{\left( 4 c^{3/2} (a - b + c) \sqrt{c + b \tan [d + e x]^2 + a \tan [d + e x]^4} \sqrt{a + \cot [d + e x]^4 (c + b \tan [d + e x]^2)} \right) +}
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{2 c^{3/2} (a-b+c) \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \tan [d+e x]^2 \sqrt{a+\cot [d+e x]^4 (c+b \tan [d+e x]^2)} \left( 2 (a-b+c) \operatorname{Csc}[d+e x] \right. \\
& \left. \operatorname{Sec}[d+e x] - \frac{2 c^{3/2} \operatorname{Sec}[d+e x]^2 \tan [d+e x]}{\sqrt{a-b+c} (1+\tan [d+e x]^2)} - \frac{a \left( 2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + \frac{\sqrt{c} (2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + 4 a \operatorname{Sec}[d+e x]^2 \tan [d+e x]^3)}{\sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \right)}{2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \right) + \\
& \frac{b \left( 2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + \frac{\sqrt{c} (2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + 4 a \operatorname{Sec}[d+e x]^2 \tan [d+e x]^3)}{\sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \right)}{2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} - \\
& \frac{c \left( 2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + \frac{\sqrt{c} (2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + 4 a \operatorname{Sec}[d+e x]^2 \tan [d+e x]^3)}{\sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \right)}{2 c+b \tan [d+e x]^2+2 \sqrt{c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} + \left( c^{3/2} \left( 2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + 2 \right. \right. \\
& \left. \left. \left( -2 a \operatorname{Sec}[d+e x]^2 \tan [d+e x] + \frac{\sqrt{a-b+c} (2 b \operatorname{Sec}[d+e x]^2 \tan [d+e x] + 4 a \operatorname{Sec}[d+e x]^2 \tan [d+e x]^3)}{2 \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4}} \right) \right) \right) / \\
& \left. \left( \sqrt{a-b+c} \left( b (-1+\tan [d+e x]^2) + 2 \left( c-a \tan [d+e x]^2 + \sqrt{a-b+c} \sqrt{c+b \tan [d+e x]^2+a \tan [d+e x]^4} \right) \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cot [d+e x]^5}{(a+b \cot [d+e x]^2+c \cot [d+e x]^4)^{3/2}} dx$$

Optimal (type 3, 160 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{2 a-b+(b-2 c) \cot [d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}} \right]}{2 (a-b+c)^{3/2} e} - \frac{a (2 a-b) + ((a-b) b+2 a c) \cot [d+e x]^2}{(a-b+c) (b^2-4 a c) e \sqrt{a+b \cot [d+e x]^2+c \cot [d+e x]^4}}$$

Result (type 4, 78272 leaves): Display of huge result suppressed!

**Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[d + e x]^3}{(a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c)\text{Cot}[d + e x]^2}{2\sqrt{a - b + c}\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}\right]}{2(a - b + c)^{3/2}e} + \frac{a(b - 2c) + (2a - b)c\text{Cot}[d + e x]^2}{(a - b + c)(b^2 - 4ac)e\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}$$

Result (type 4, 78265 leaves): Display of huge result suppressed!

**Problem 30: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[d + e x]}{(a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c)\text{Cot}[d + e x]^2}{2\sqrt{a - b + c}\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}\right]}{2(a - b + c)^{3/2}e} - \frac{b^2 - 2ac - bc + (b - 2c)c\text{Cot}[d + e x]^2}{(a - b + c)(b^2 - 4ac)e\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}$$

Result (type 4, 78291 leaves): Display of huge result suppressed!

**Problem 31: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[d + e x]}{(a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 280 leaves, 12 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a + b\text{Cot}[d + e x]^2}{2\sqrt{a}\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}\right]}{2a^{3/2}e} - \frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c)\text{Cot}[d + e x]^2}{2\sqrt{a - b + c}\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}\right]}{2(a - b + c)^{3/2}e} - \frac{b^2 - 2ac + bc\text{Cot}[d + e x]^2}{a(b^2 - 4ac)e\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}} + \frac{b^2 - 2ac - bc + (b - 2c)c\text{Cot}[d + e x]^2}{(a - b + c)(b^2 - 4ac)e\sqrt{a + b\text{Cot}[d + e x]^2 + c\text{Cot}[d + e x]^4}}$$

Result (type 4, 181078 leaves): Display of huge result suppressed!

### Problem 32: Humongous result has more than 200000 leaves.

$$\int \frac{\text{Tan}[d + e x]^3}{(a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4)^{3/2}} dx$$

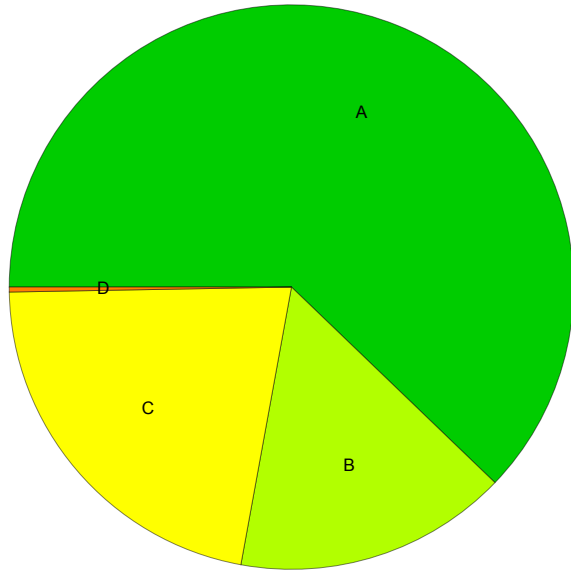
Optimal (type 3, 478 leaves, 16 steps):

$$\begin{aligned} & - \frac{\text{ArcTanh}\left[\frac{2 a + b \text{Cot}[d + e x]^2}{2 \sqrt{a} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2 a^{3/2} e} - \frac{3 b \text{ArcTanh}\left[\frac{2 a + b \text{Cot}[d + e x]^2}{2 \sqrt{a} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{4 a^{5/2} e} + \frac{\text{ArcTanh}\left[\frac{2 a - b + (b - 2 c) \text{Cot}[d + e x]^2}{2 \sqrt{a - b + c} \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}}\right]}{2 (a - b + c)^{3/2} e} + \\ & \frac{b^2 - 2 a c + b c \text{Cot}[d + e x]^2}{a (b^2 - 4 a c) e \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}} - \frac{b^2 - 2 a c - b c + (b - 2 c) c \text{Cot}[d + e x]^2}{(a - b + c) (b^2 - 4 a c) e \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}} - \\ & \frac{(b^2 - 2 a c + b c \text{Cot}[d + e x]^2) \text{Tan}[d + e x]^2}{a (b^2 - 4 a c) e \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4}} + \frac{(3 b^2 - 8 a c) \sqrt{a + b \text{Cot}[d + e x]^2 + c \text{Cot}[d + e x]^4} \text{Tan}[d + e x]^2}{2 a^2 (b^2 - 4 a c) e} \end{aligned}$$

Result (type ?, 293 889 leaves): Display of huge result suppressed!

## Summary of Integration Test Results

357 integration problems



A - 222 optimal antiderivatives

B - 56 more than twice size of optimal antiderivatives

C - 78 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts